Performance Evaluation: Anti-windup Two-Loop PID Controller for Rotary Single Inverted Pendulum

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Introduction

Rotary Single Inverted Pendulum (RSIP) is an example of non minimum phase and highly unstable system. It has a variety of applications in the real world e.g. military appliances (missile launching pads), satellite launching system, heavy cranes, segway, robotics etc. For the controller design, Linear Quadratic Regulator (LQR) technique ensures the loop robustness and meet design constraints such as tracking set-point, voltage limits and pendulum position. Two loop PID controller is designed via pole placement, where the desired close loop poles are obtained from LQR technique. It is seen that the robustness of two loop PID controller is higher than LQR design.

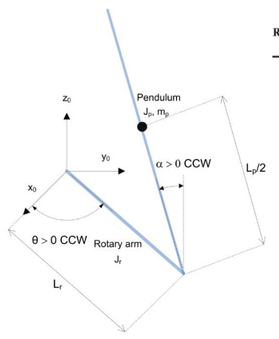
In this project LQR, State Observer feedback and two-loop PID controller for RSIP are successfully designed and implemented. Real time experimental demonstration with performance evaluation in terms of transient response and robustness has been done, which validates the superiority of two-loop PID controller for RSIP.

Modelling

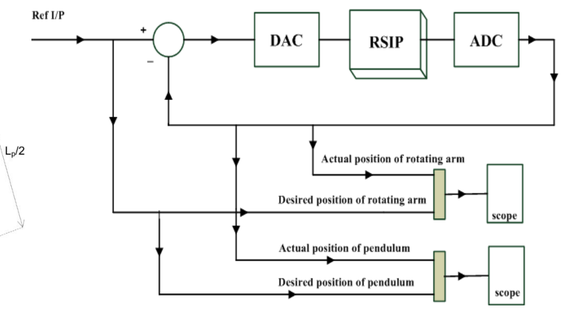
Experimental arrangement of RSIP is provided by Quanser (SRV-02), interfaced by data acquisition card and driven by MATLAB-Simulink based on real-time software (QUARC).



(a) Laboratory Setup



(b) Free Body Diagram of RSIP



(c) Schematic Diagram

Based on the configuration of the SRV02 and the RSIP system, the state space representation of the system is,

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 52.78 & -0.66 & -0.65 \\ 0 & 97.98 & -0.65 & -1.21 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 274.45 \\ 273.92 \end{bmatrix} v_i \quad y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_i$$

Transfer function of Rotational Arm (θ) and Pendulum Position (α) is given by (2) and (3) respectively,

$$\frac{\theta(s)}{v_i(s)} = \frac{274.5s^2 + 154s + 12430}{s^4 + 1.87s^3 - 97.61s^2 - 26.69s} = P_1 \qquad (2)$$

$$\frac{\alpha(s)}{v_i(s)} = \frac{273.9s^2 + 2.39s}{s^4 + 1.87s^3 - 97.61s^2 - 26.69s} = P_2 \qquad (3)$$

Controller Design Methods

1) Controller design using LQR Technique

Linear Quadratic Regulator (LQR) is an optimal state feedback control strategy, which regulates the control signal such that objective is achieved with take care of constraints. If the system is perturbed, the control input is regulated such that pendulum is balanced in vertical position and rotating arm tracks the set-point with less control energy and criteria *J* minimized.

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u) dt$$

Different combination of Q and R are tested and LQR shows better satisfactory response for

$$Q = diag(1, 100, 0, 0) \quad and \quad R = 10$$

Gain matrix (*k*) is given by : $k = [-0.31623 \quad 4.5308 \quad -0.24353 \quad 0.40985]$

Eigen values of state feedback matrix *(A – Bk)* : $s = -22.0779 \pm j19.7108 \quad and \quad s = -1.5705 \pm j1.422$

Characteristic Equation : $s^4 + 47.28s^3 + 1019s^2 + 2949s + 3932 = 0$

2) Design of State Observer Feedback Controller

In the pole placement approach, it is assumed that all state variables are available for feedback. But in reality, all states may not be available. So estimation of unmeasurable states is required; a program that estimates the various states is known as observer.

Observer gain matrix (*L*) is given by : $L = \begin{bmatrix} 2207.7 & 0 & 1 & 0 \\ 0 & 2207.7 & 0 & 1 \\ 0 & 52.8 & 156.4 & -0.6 \\ 0 & 98 & -0.6 & 155.8 \end{bmatrix}$

3) Design of Two-Loop PID Controller

Schematic diagram of controller scheme with RSIP is shown. For maintaining the robustness property of LQR in two loop PID controller, the same closed loop poles are selected. In order to stabilize the pendulum in vertical position and control the rotating arm (for set-point tracking), two loop PID controller is used.

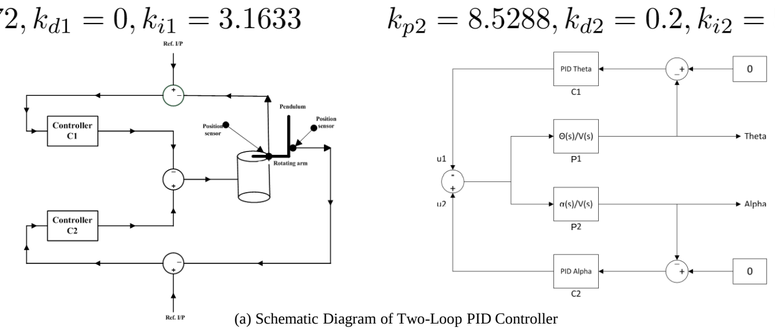
Two Controllers are given by : $C_1(s) = k_{p1} + k_{d1}s + \frac{k_{i1}}{s}$ and $C_2(s) = k_{p2} + k_{d2}s + \frac{k_{i2}}{s}$

Characteristic equation of control scheme is given by : $1 - P_1C_1 + P_2C_2 = 0$

Adding the pole at six times of dominant pole (*s + 10*) in characteristic equation. New characteristic equation is given by : $s^5 + 57.28s^4 + 1491.8s^3 + 13139s^2 + 33422s + 39320 = 0$

On comparing both the characteristic equations, PID controller gains are given by :

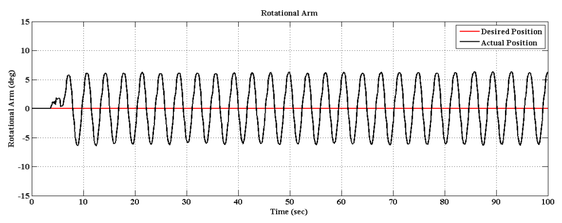
$k_{p1} = 2.72, k_{d1} = 0, k_{i1} = 3.1633 \qquad k_{p2} = 8.5288, k_{d2} = 0.2, k_{i2} = 52.76$



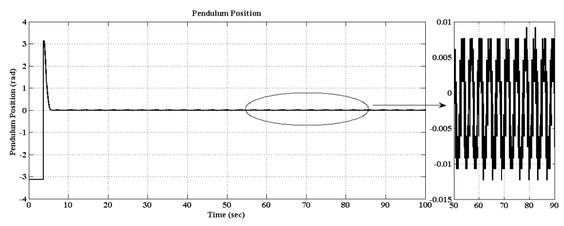
(a) Schematic Diagram of Two-Loop PID Controller

Experimental Results

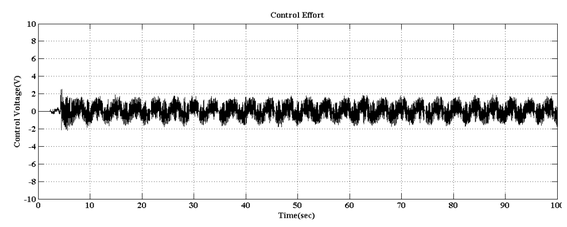
State Observer



(a) Variation in Rotational Arm

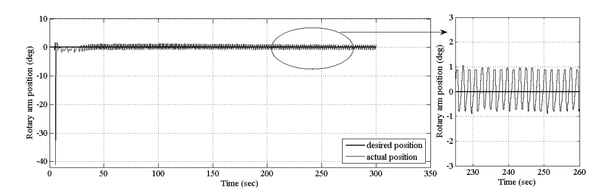


(b) Variation in Pendulum Position

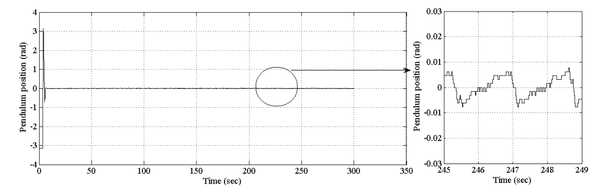


(c) Variation in Control effort

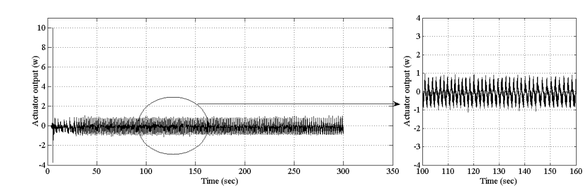
Two loop PID



(a) Variation in Rotational Arm



(b) Variation in Pendulum Position

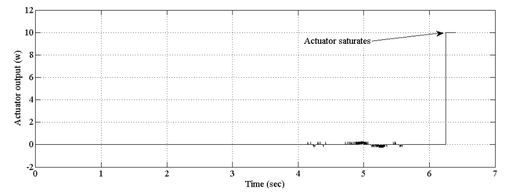


(c) Variation in Control effort

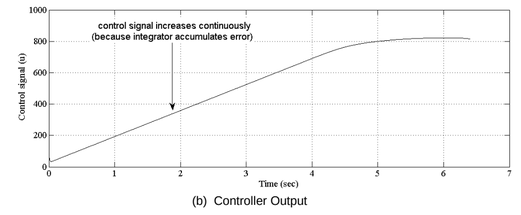
Parameter Description	LQR	State-observer	PID
Rotational Arm (deg)	±5	±6	±1
Pendulum Position (rad)	±0.015	±0.01	±0.006
Control Effort (V)	±1.8	±1.3	±1

Table 1 Comparative analysis of Various Controllers

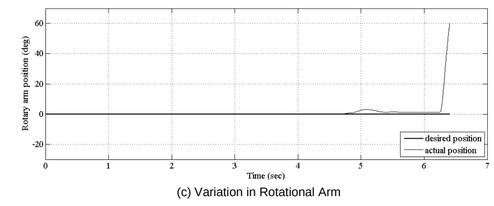
Windup Phenomena



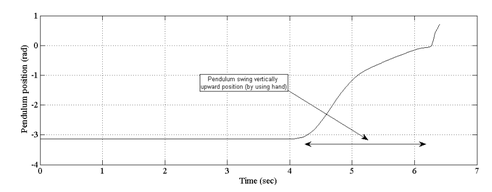
(a) Actuator Output



(b) Controller Output



(c) Variation in Rotational Arm



(d) Variation in Pendulum Position

Anti-Windup Technique :

Practically, RSIP have electronic circuitry components which have maximum and minimum limit of voltage constraints. Whenever control signal is beyond the voltage constraints, actuator saturates and integrator accumulates the error thus generating large control signal, this phenomena is known as integral windup.

To overcome this problem, conditional integration (known as clamping integral) anti-windup technique is used. In this technique, integrator will turn on when control signal is in linear region of actuator otherwise it turns off.

Conclusion :

Proposed two loop anti-windup PID controller has been experimentally illustrated for Rotary Single Inverted Pendulum (RSIP). Drawbacks of State-observer and LQR state-feedback are that the system becomes sluggish and initially deviation is more in set-point tracking respectively; but with proposed topology these drawbacks are overcame.

Table I incorporates the comparative analysis of various controllers with RSIP.

For future scope, two loop anti-windup Fractional order PID controller can be applied to RSIP for better set-point tracking compares to two loop anti-windup Integer order PID controller