B.TECH PROJECT REPORT
ON

IMPLEMENTATION OF DIFFERENT CONTROLLERS ON ROTARY INVERTED PENDULUM

Thesis submitted to Visvesvaraya National Institute of Technology, Nagpur
In partial fulfilment of requirement for the award of degree of
BACHELOR OF TECHNOLOGY
(ELECTRICAL AND ELECTRONICS ENGINEERING)

Submitted By

KANDUKURI VISHNU SAI	(BT12EEE030)
KISHORE SESHADRI	(BT12EEE035)
PAVAN SRIKANTH	(BT12EEE048)
ROHIT SALEM	(BT12EEE063)
SUDHEER KUMAR	(BT12EEE065)
SHUBHAM M WAGH	(BT12EEE099)

Under the Guidance of

DR. A. S. JUNGHARE



DEPARTMENT OF ELECTRICAL ENGINEERING
VISVESVARAYA NATIONAL INSTITUTE OF TECHNOLOGY
NAGPUR

2015 - 2016



Visvesvaraya National Institute of Technology, Nagpur
2015 - 2016

CERTIFICATE

This is to certify that the project work entitled **"Implementation of different controllers on Rotary Inverted Pendulum"**, is a bonafide work by **Mr. Kandukuri Vishnu Sai, Mr. Kishore Seshadri, Mr. Pavan Srikanth, Mr. Rohit Salem, Mr. Sudheer Kumar and Mr. Shubham M Wagh** in the Department of Electrical Engineering, Visvesvaraya National Institute of Technology, Nagpur, in partial fulfilment of the requirements of the award of the degree of Bachelor of Technology in Electrical and Electronics Engineering.

Prof. Dr. A.S Junghare
Mentor and Coordinator
Associate Professor
Department of Electrical Engg.
VNIT, Nagpur

Prof. Dr. S.R Bhide
Head of Department
Department of Electrical Engg.
VNIT, Nagpur



Visvesvaraya National Institute of Technology, Nagpur
2015 - 2016

DECLARATION

This is to declare that the project work entitled **"Implementation of different controllers on rotary inverted pendulum"**, is a bonafide work performed by us, the below mentioned students. This project work is being submitted and forwarded in partial fulfilment of the requirements for the award of the degree of Bachelor of Technology in Electrical and Electronics Engineering from Visvesvaraya National Institute of Technology, Nagpur.

To the best of our knowledge this project report has not been submitted to any other institution or university.

Vishnu Kandukuri	Kishore Seshadri	Pavan Srikanth
Sudheer Kumar	Rohit Salem	Shubham M Wagh

Acknowledgement

"The finest task of an Engineer - to survey, to plan, to suggest resources and material, assemble them for task in hand, implement and procedure results"

- Sir. Mokshagundam Visvesvaraya

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Abstract

The Inverted Pendulum is a classical but interesting problem in the field of control theory. This thesis focusses on the system design, stability analysis and modelling of the rotary single inverted pendulum. It also provides the steps to design the controllers for the SRV02 series Quanser rotary set-up. The controllers used include State Feedback using pole placement, State Observer and PID controller after the state parameters of the system was optimised using the Linear Quadratic Regulated (LQR) technique. The study includes both simulation and experimental results which leads to a comparative analysis of the robustness of the system under the three controllers.

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Chapter 1

Introduction

1.1 Single Inverted Pendulum

The inverted pendulum is a classic problem in dynamics and control theory and is used as a benchmark for testing control strategies (PID controllers, state space representation, neural networks, fuzzy control, genetic algorithms, etc.)[1] The designing of a controller for stabilization of pendulum is a challenging task. This design becomes complex because of the physical limitations on applied voltage, rotation angle and pendulum angle. Basically the system has two degrees of freedom with single control input. Also, it is a non-minimal phase system which makes it unstable [2]. Several researchers have investigated inverted pendulum system to address these issues [3- 8]. Inverted pendulum was introduced as a cart type. However, to compensate the limitation of cart length, rotary inverted pendulum was invented [9]. Segway human transporter, movement of human arm, walking of biped animals, launching a missile, and movement of robotic arm are examples of inverted pendulum [10]. In the past, it is recognized that the conventional PID/PD controller has governed the control industry. It is largely because of the simplicity of its standard structure and the past record for efficient solution when properly tuned and installed. It takes care of the transient as well as steady state response with three terms P, I, and D. The parameter values of PID can be evaluated by any tuning method mentioned in literature. But there is no assurance of loop robustness and no guarantee to fulfill the design constraints. In literature, LQR is reported as an optimal controller which provides robustness as well as fulfills the constraints requirement [11-12].To achieve better transient and steady state response with robustness, a two loop PID controller using the desired characteristic equation is designed [13].

In this project LQR, Full state feedback observer and dual-loop PID controller for a rotary inverted pendulum are successfully designed and implemented. Real time experimental demonstration with performance evaluation in terms of transient response and robustness has been done, which validates the superiority of dual loop PID controller for rotary inverted pendulum.

1.2 Scope of the thesis

This thesis primarily focusses on the complete mathematical modelling of the system, design of the controller for the balancing control of the pendulum and a comparative study of the following three controllers, along with the simulation and practical implementation results:

(a) State feedback using LQR optimisation.

(b) State observer feedback controller

(c) Dual - loop PID controller

The thesis also provides a novel method of implementing the state observer feedback control and the PID controller for the balance control which has been discussed in greater details in subsequent chapters.

Chapter 2

Mathematical Modelling

2.1 Introduction

When approaching the solution of a control problem, the mathematical modelling forms the basis for many modern control strategies.

The more that is known about a dynamic system, the more accurate a mathematical model can be. Accurate mathematical modelling allows the design of faster, more accurate and effective controllers. This is because mathematical models allow design, test and development of controllers rapidly using MATLAB even before the physical rig is constructed.

Accurate mathematical models[17] of the pendulum system can be developed, provided that there is not excessive stiction (static friction), backlash or bearing slack present. The fore mentioned non linearity's make accurate modeling and control of the system much more difficult.

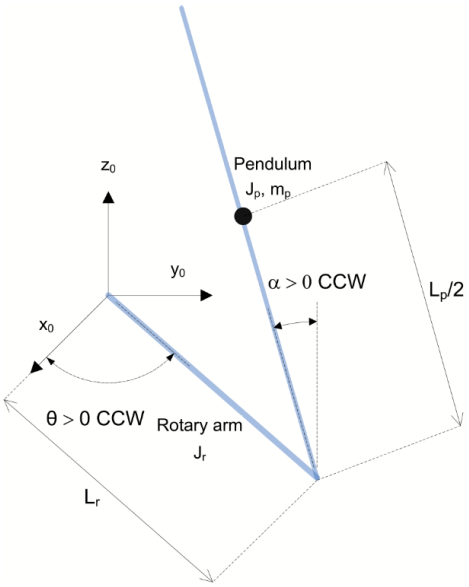
2.2 Model Conventions

The free body diagram is shown in the following figure 2.1:

The rotary arm is pivoted with a servo motor(SRV02) system and is actuated to balance the pendulum. Various pendulum specifications are shown in table (2.1)



(a) Quanser Rotary Inverted Pendulum



(b) Free body diagram of rotary inverted pendulum

Figure 2.1: Rotary Inverted Pendulum

Variable	Value	Unit
m_p	0.127	kg
L_p	0.337	m
J_p	0.0012	kg.m^2
B_p	0.0024	N.m.s/rad
m_r	0.257	kg
L_r	0.2519	m
J_r	9.98x10$^-$4	kg.m^2
R_m	2.6	Ω
K_g	70	
k_m	0.0077	V/(rad/s)
k_t	0.0077	N-m/A
g	9.8	m/s^2
η_g	0.9	
η_m	0.69	

Table 2.1: Rotary Inverted pendulum specification

2.3 Servomotor Model

The servomotor model was developed with the following block diagram[19] as shown in Figure 2.2

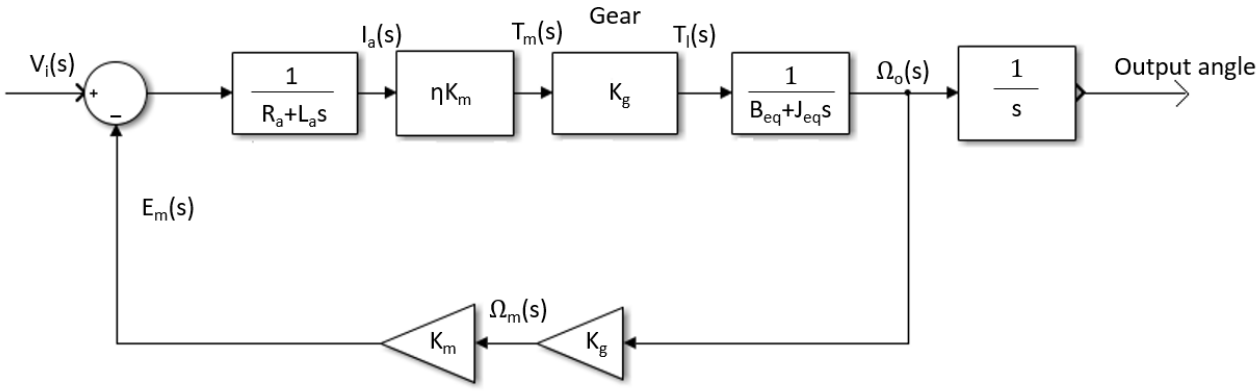


Figure 2.2: Servo plant Block Diagram

$$\frac{\theta_0(s)}{V_i(s)} = \frac{\frac{\eta K_m K_g}{R_a J_{eq}}}{s\left(s + \frac{B_{eq}}{J_{eq}} + \frac{\eta K_m^2 K_g^2}{R_a J_{eq}}\right)} \tag{2.1}$$

or

$$\frac{\theta_0(s)}{V_i(s)} = \frac{a_m}{s(s + b_m)} \tag{2.2}$$

where

$$a_m = \frac{\eta K_m K_g}{R_a J_{eq}}, \quad b_m = \frac{B_{eq}}{J_{eq}} + \frac{\eta K_m^2 K_g^2}{R_a J_{eq}} \tag{2.3}$$

Also from the block diagram the s-domain output torque $T_L(s)$ is

$$T_L(s) = \frac{\eta K_m K_g}{R_a}[V_i(s) - K_m K_g \Omega_0(s)] \tag{2.4}$$

Therefore the expression for the output torque in time-domain is

$$T_L(t) = K_1 v_i(t) - K_2 \omega(t) \tag{2.5}$$

where

$$K_1 = \frac{\eta K_m K_g}{R_a}, \quad K_2 = \frac{\eta K_m^2 K_g^2}{R_a} \tag{2.6}$$

In this project the servomotor is arranged for the high gear ratio as shown in Figure 2.3 . For this configuration the gear ratio is $K_g = (14)(5)$

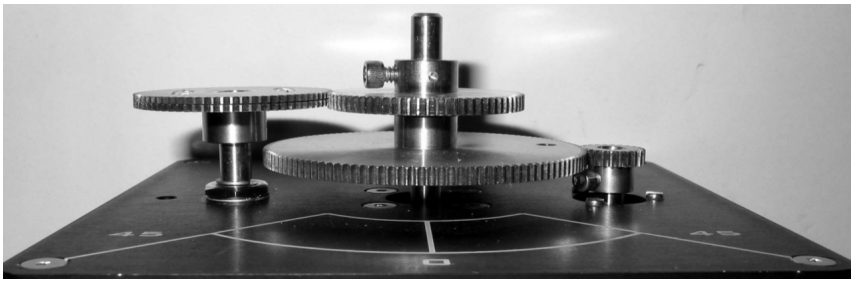


Figure 2.3: High gear ratio configurations

2.4 Inverted Pendulum Model

2.4.1 Using Lagrangian Formulation

Lagrange method[5] is used to find the equations of motion of the system. The motions of the rotary arm and the pendulum with respect to servo motor voltage is described by these EOMs.

$$\frac{\partial^2 L}{\partial t \partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i \tag{2.7}$$

The variables q_i are called the generalized coordinates. For this system let

$$q(t)^T = \begin{bmatrix} \theta(t) & \alpha(t) \end{bmatrix} \tag{2.8}$$

With the generalized coordinates defined, the Euler-Lagrange equations for the rotary inverted pendulum system are

$$\frac{\partial^2 L}{\partial t \partial \dot{\theta}} - \frac{\partial (L)}{\partial \theta} = Q_1 \tag{2.9}$$

$$\frac{\partial^2 L}{\partial t \partial \dot{\alpha}} - \frac{\partial (L)}{\partial \alpha} = Q_2 \tag{2.10}$$

The Lagrangian of the system is described as

$$L = T - V \tag{2.11}$$

where T is the total kinetic energy and V is the total potential energy of the system.

The generalized forces Q_i are used to describe non-conservative forces(e.g. friction) applied to the system with respect to generalized co-ordinates. For this case non-conservative force acting on the rotary arm is

$$Q_1 = \tau - B_r\dot{\theta} \tag{2.12}$$

and acting on the pendulum is

$$Q_2 = -B_p\dot{\alpha} \tag{2.13}$$

After the Lagrangian of the system is found and various derivatives are calculated to find the following EOMs from equations (2.9) and (2.10)

$$\Big(m_pL_r^2 + \frac{1}{4}m_pL_p^2\cos(\alpha)^2 + J_r\Big)\ddot{\theta} - \Big(\frac{1}{2}m_pL_pL_r\cos(\alpha)\Big)\ddot{\alpha} \tag{2.14}$$

$$+\Big(\frac{1}{2}m_pL_p^2\sin(\alpha)\cos(\alpha)\Big)\dot{\theta}\dot{\alpha} + \Big(\frac{1}{2}m_pL_pL_r\sin(\alpha)\Big)\dot{\alpha}^2 = \tau - B_r\dot{\theta} \tag{2.15}$$

$$-\frac{1}{2}m_pL_pL_r\cos(\alpha)\ddot{\theta} + \Big(J_p + \frac{1}{4}m_pL_p^2\Big)\ddot{\alpha} - \frac{1}{4}m_pL_p^2\cos(\alpha)\sin(\alpha)\dot{\theta}^2 \tag{2.16}$$

$$-\frac{1}{2}m_pL_pg\sin(\alpha) = -B_p\dot{\alpha} \tag{2.17}$$

The torque applied at the base of the rotary arm is generated by a servo motor which is described by the equation

$$\tau = \frac{\eta_gK_g\eta_mk_t(V_m - K_gK - m\dot{\theta})}{R_m} \tag{2.18}$$

2.4.2 Linearizing

Initial condition for all the variables are taken zero. $\theta = 0°$, $\alpha = 0°$, $\dot{\theta} = 0$, $\dot{\alpha} = 0$ for a multivariable system control variable z is defined

$$z^T = \begin{bmatrix} z_1 & z_2 \end{bmatrix} \tag{2.19}$$

and if $f(z)$ is to be linearized about the operating point

$$z_0^T = \begin{bmatrix} a & b \end{bmatrix} \tag{2.20}$$

the linearized function can be written as

$$f_{lin} = f(z_0) + \Big(\frac{\partial f(z)}{\partial z_1}\Big)\Big|_{z=z_0}(z_1 - a) + \Big(\frac{\partial f(z)}{\partial z_2}\Big)\Big|_{z=z_0}(z_2 - b) \tag{2.21}$$

Applying this linearization method and taking the left hand side of equations (2.14) and (2.16) as $f(z)$

we get the linearized form of the EOMs.

$$(m_p L_r^2 + J_r)\ddot{\theta} - \frac{1}{2}m_p L_p L_r \ddot{\alpha} = \tau - B_r\dot{\theta} \tag{2.22}$$

$$-\frac{1}{2}m_p L_p L_r \ddot{\theta} + \Big(J_p + \frac{1}{4}m_p L_p^2\Big)\ddot{\alpha} - \frac{1}{2}m_p L_p g\alpha = -B_p\dot{\alpha} \tag{2.23}$$

Equations (2.22) and (2.23) can be written in matrix form as follows

$$\begin{bmatrix} m_p L^2 r + J_r & -\frac{1}{2}m_p L_p L_r \\ -\frac{1}{2}m_p L_p L_r & J_p + \frac{1}{4}m_p L_p^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} B_r & 0 \\ 0 & B_p \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{2}m_p L_p g\alpha \end{bmatrix} = \begin{bmatrix} \tau \\ 0 \end{bmatrix} \tag{2.24}$$

2.4.3 State Space Model of the System

The linear state space equations are

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{2.25}$$

$$y = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \tag{2.26}$$

here $\mathbf{x}$ is the state and u is the control input. $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are the state-space matrices. For the rotary inverted pendulum system the state and the output are defined

$$x^T = \begin{bmatrix} \theta & \alpha & \dot{\theta} & \dot{\alpha} \end{bmatrix} \tag{2.27}$$

and

$$y^T = \begin{bmatrix} \theta & \alpha \end{bmatrix} \tag{2.28}$$

In the output equation only the position of the servo and link angles are being measured. Based on this **C** and **D** matrices of the output equations are

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{2.29}$$

and

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2.30}$$

From equation (2.24)

$$\begin{bmatrix} m_p L^2 r + J_r & -\frac{1}{2} m_p L_p L_r \\ -\frac{1}{2} m_p L_p L_r & J_p + \frac{1}{4} m_p L_p^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} \tau - B_r \dot{\theta} \\ -\frac{1}{2} m_p L_p g \alpha - B_p \dot{\alpha} \end{bmatrix} \tag{2.31}$$

solving for the acceleration terms-

$$\begin{bmatrix} \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \frac{1}{J_T} \begin{bmatrix} J_p + \frac{1}{4} m_p L_p^2 & \frac{1}{2} m_p L_p L_r \\ \frac{1}{2} m_p L_p L_r & m_p L^2 r + J_r \end{bmatrix} \tag{2.32}$$

here, $J_T = (J_p m_p L_r^2 + J_r J_p + \frac{1}{4} J_r m_p L^2 p)$ we find the following acceleration expressions from the above

$$\ddot{\theta} = \frac{1}{J_T}\Big(-\Big(J_p + \frac{1}{4} m_p L^2 p\Big) B_r \dot{\theta} - \frac{1}{2} m_p L_p L_r B_p \dot{\alpha} + \frac{1}{4} m_p^2 L_p^2 L_r g \alpha + \Big(J_p + \frac{1}{4} m_p L^2 p\Big)\tau \Big) \tag{2.33}$$

$$\ddot{\alpha} = \frac{1}{J_T}\Big(\frac{1}{2} m_p L_p L_r B_p \dot{\theta} - (J_r + m_p L_r^2) B_p \dot{\alpha} + \frac{1}{2} m_p L_p g (J_r + m_p L_r^2) \alpha + m_p L_p L_r \tau \Big) \tag{2.34}$$

from (2.27) $\dot{x_1} = x_3$ and $\dot{x_2} = x_4$ substituting **x** in the acceleration terms the space matrices **A** and **B** can be calculated

$$A = \frac{1}{J_T}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{4}m_p^2 L_p^2 L_r g & -(J_p + \frac{1}{4}m_p L_p^2)B_r & -\frac{1}{2}m_p L_p L_r B_p \\ 0 & \frac{1}{2}m_p L_p g(J_r + m_p L^2 r) & \frac{1}{2}m_p L_p L_r B_r & -(J_r + m_p L_r)B_p \end{bmatrix} \tag{2.35}$$

$$B = \frac{1}{J_T}\begin{bmatrix} 0 \\ 0 \\ J_p + \frac{1}{4}m_p L_p^2 \\ \frac{1}{2}m_p L_p L_r \end{bmatrix} \tag{2.36}$$

Based on the typical configuration of the SRV02 and the rotary inverted pendulum system, the above state space representation of the system is

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 52.78 & -0.66 & -0.65 \\ 0 & 97.98 & -0.65 & -1.21 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 274.45 \\ 273.92 \end{bmatrix} v_i \tag{2.37}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_i \tag{2.38}$$

2.4.4 Transfer Function Model

Based on the state space equations (2.37) and (2.38), it is possible to derive the following two transfer functions.

$$\frac{\theta(s)}{v_i(s)} = \frac{274.5s^2 + 154s + 12440}{s^4 + 1.87s^3 - 97.61s^2 - 30.37s} \tag{2.39}$$

$$\frac{\alpha(s)}{v_i(s)} = \frac{273.9s}{s^3 + 1.87s^2 - 97.61s - 30.37} \tag{2.40}$$

Chapter 3

Conventional Control System Analysis

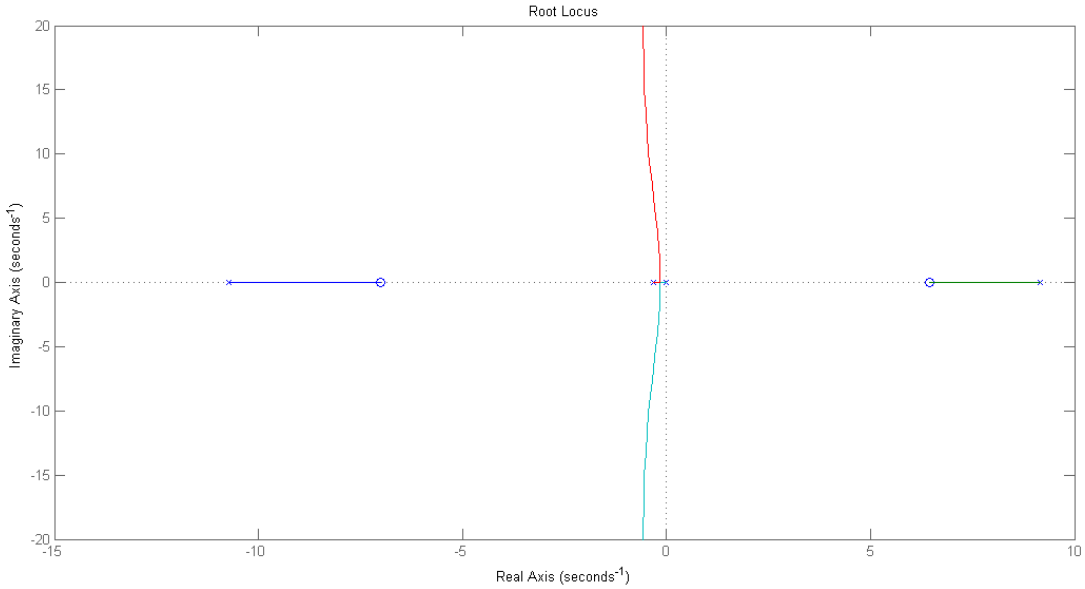
As followed in most of the literatures, the analysis of rotary inverted pendulum begins with the conventional control system analysis using the root - locus method and frequency response of the system. Since the current system is a fourth order system, the analysis is done directly using the MATLAB software.

3.1 Degrees of Freedom

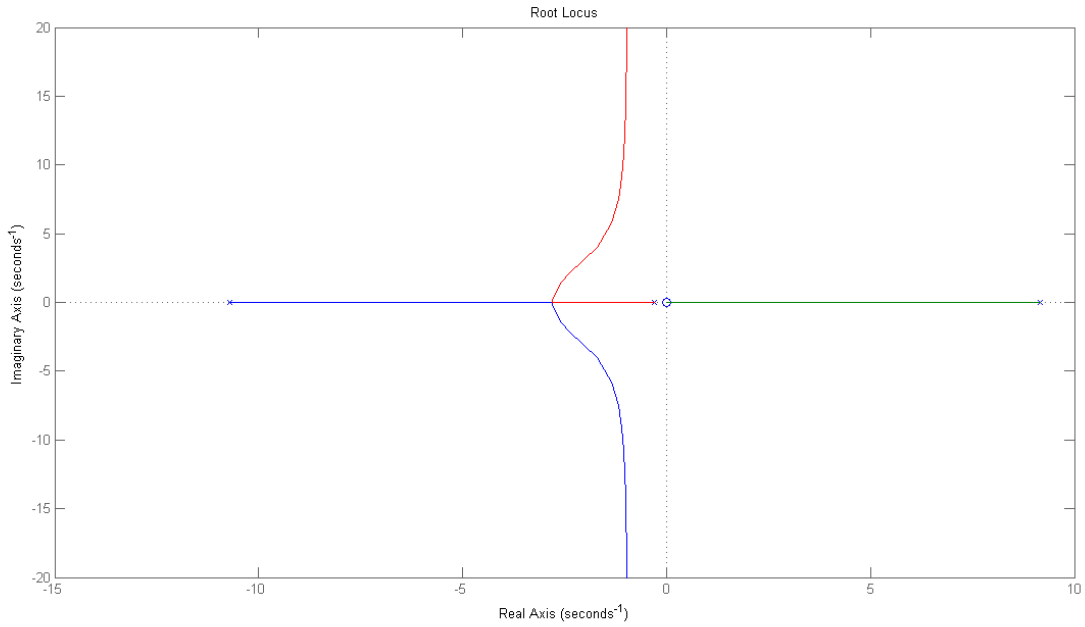
In physics, the degree of freedom (DOF) of a mechanical system is the number of independent parameters that determine the independent parameters that define its configuration. To a control engineer[18], the degrees of freedom is defined as the number of independent closed loop transfer functions in a control system. The single inverted pendulum may have one or two degrees of freedom. It has one degree of freedom if the controlling motor is placed at the pivoting point (as in the case of the cart and pole type system) while it has two degrees of freedom when the pendulum is controlled by the motor at the end of the arm (the rotary inverted pendulum). The degrees of freedom, in a way, helps the control engineer in deciding the state parameters for the system. The two degrees of freedom involved in the rotary inverted pendulum is the horizontal angle of the arm (θ) and the vertical angle of the pendulum (α).

3.2 Root - Locus Method

As Ogata[19] rightly states: The basic characteristics of the transient response of a closed-loop system is closely related to the location of the closed-loop poles. If the system has a variable loop gain, then the location of the closed-loop poles depends on the value of the loop-gain chosen. It is important, therefore, that the designer knows how the closed-loop poles move in the s-plane as the loop gain is varied. This trajectory of the closed loop pole in the s-plane with varying loop gain is termed as the root locus plot, developed by W.R.Evans. Closed-loop poles are the roots of the characteristic equation. By using the root locus method, the designer can predict the effects on the location of the closed-loop poles of varying the gain value or adding open-loop poles and/or open-loop zeros. From (3.1) it is observed that one pole each of the open loop transfer function of θ and α i.e the two defining transfer functions of the plant lies on the right hand side of the s-plane. This shows that the system is inherently unstable but the system parameters are completely controllable as will be described in the later sections. After applying the different controller schemes all the poles of the system are shifted to the left half of the S plane, hence the system is stablised as shown in figure (3.2). This provides us with the motivation of designing the controllers for the inverted pendulum system.

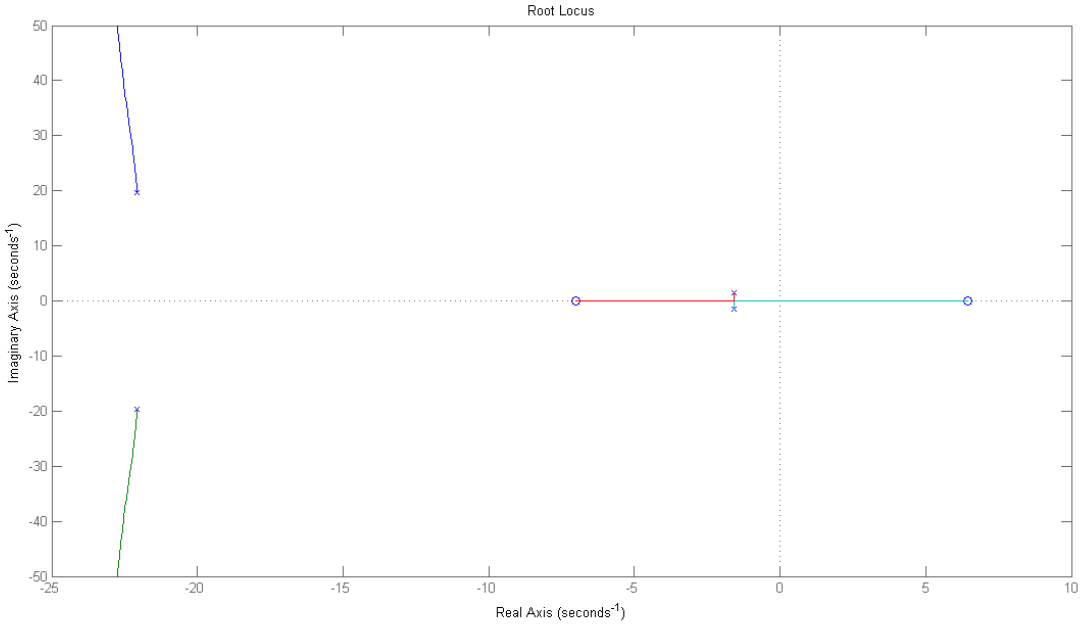


(a) Root locus plot of uncompensated $\frac{\theta(s)}{v_i(s)}$

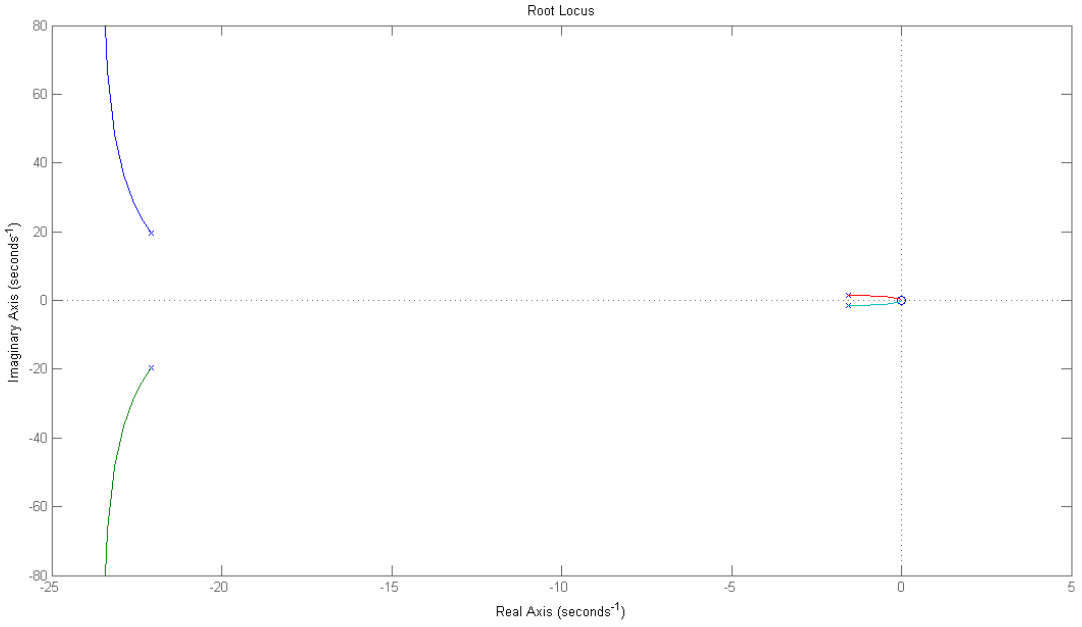


(b) Root locus plot of uncompensated $\frac{\alpha(s)}{v_i(s)}$

Figure 3.1: Root Locus plot of the uncompensated system



(a) Root locus plot of compensated $\frac{\theta(s)}{v_i(s)}$



(b) Root locus plot of compensated $\frac{\alpha(s)}{v_i(s)}$

Figure 3.2: Root Locus plot of the compensated system

3.3 Frequency Response

By the term frequency response[19], the steady-state response of a system to a sinusoidal input is studied. In frequency-response methods, the frequency of the input signal is varied over a certain range and the resulting response is studied. The frequency response and root-locus approaches complement each other. One advantage of the frequency response approach is that the data obtained from measurements on the physical system can be used without deriving its mathematical model.

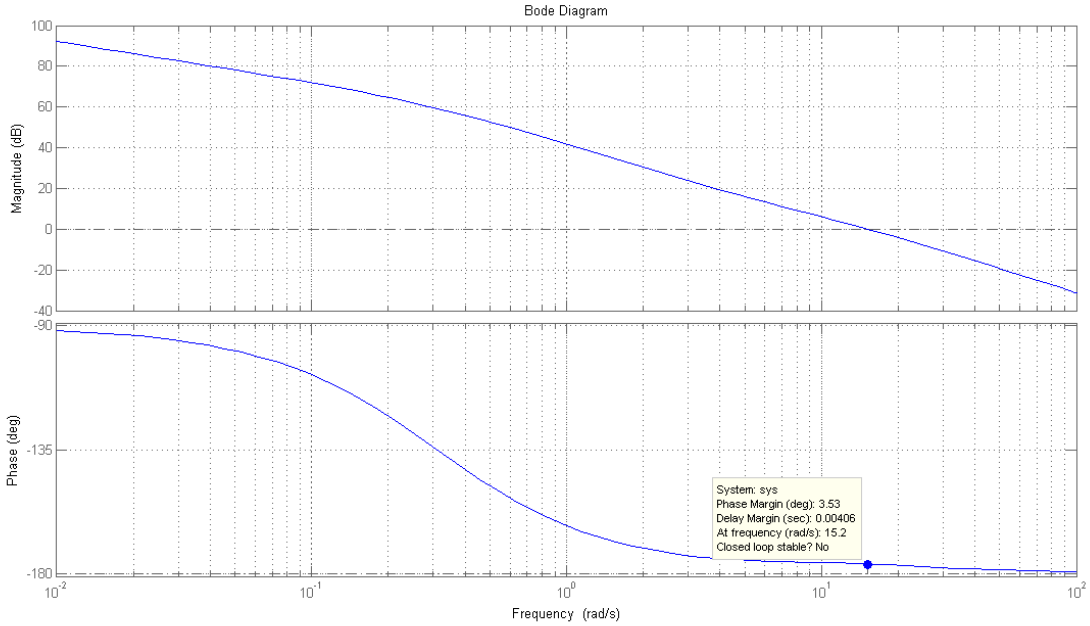
The sinusoidal transfer function, a complex function of the frequency ω, is characterized by its magnitude and phase angle, with frequency as the parameter. There are three commonly used representations of sinusoidal transfer functions:

(a) Bode diagram or logarithmic plot.

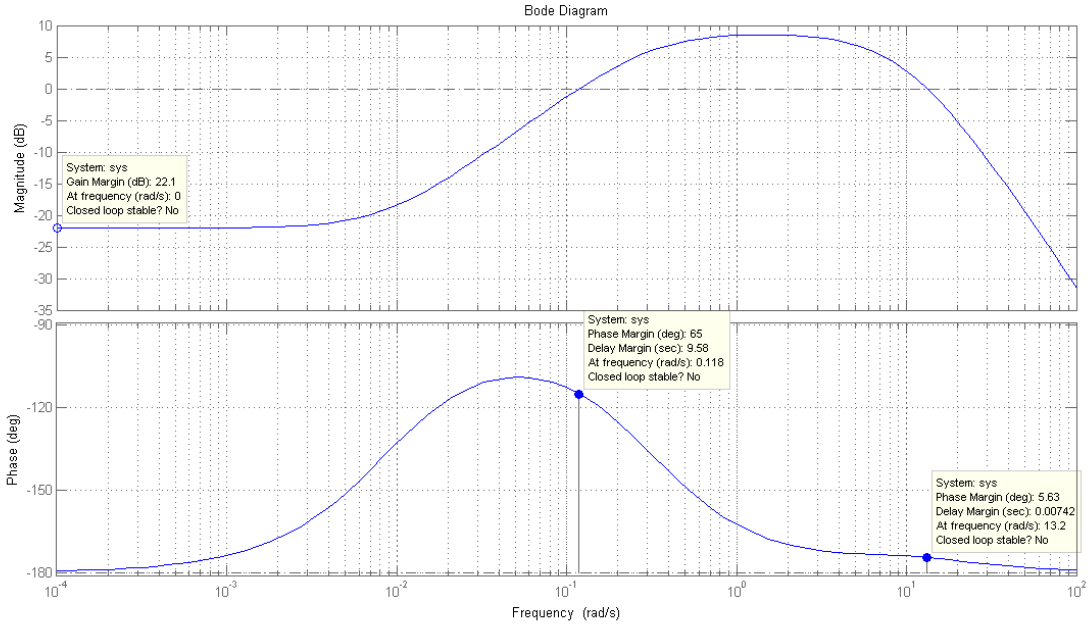
(b) Nyquist plot or polar plot

(c) Nichols plot

The Bode diagram for the horizontal arm angle θ and vertical pendulum angle α of the inverted pendulum is as illustrated in figure 3.3a and 3.3b respectively. It consists of two graphs: one is a plot of the logarithm of the magnitude of the sinusoidal transfer function; the other is a plot of the phase angle; both are plotted against the frequency on a logarithmic scale. From the Bode plot shown in figure 3.3 and figure 3.4, it is seen that the system is unstable. Unstable system is stabilised through the controllers which can be validated through the bode plot, In which gain margin and phase margin are coming out to be $-10dB$ and $28deg$ respectively

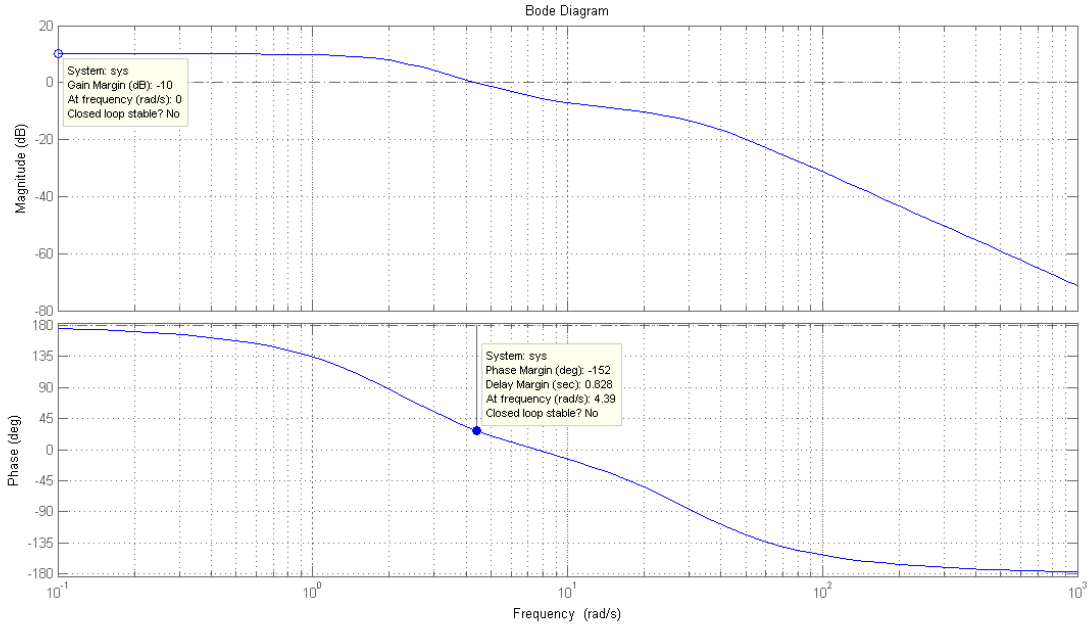


(a) Bode plot of uncompensated $\frac{\theta(s)}{v_i(s)}$

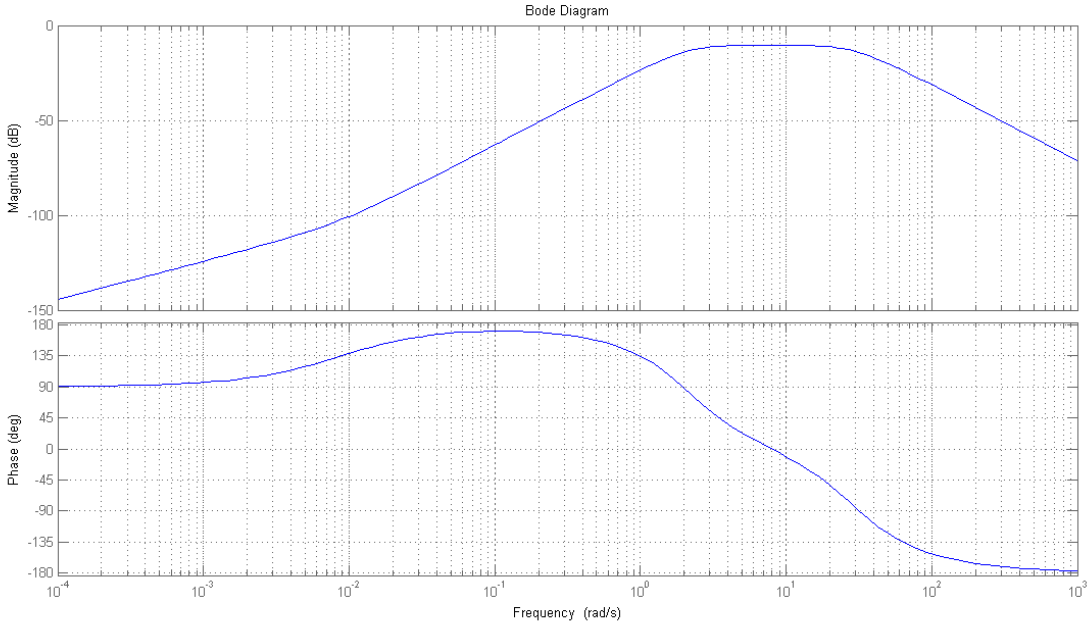


(b) Bode plot of uncompensated $\frac{\alpha(s)}{v_i(s)}$

Figure 3.3: Bode plot of the uncompensated system



(a) Bode plot of compensated $\frac{\theta(s)}{v_i(s)}$



(b) Bode plot of compensated $\frac{\alpha(s)}{v_i(s)}$

Figure 3.4: Bode plot of the sompensated system

Chapter 4

Modern Control System Analysis

The next method to analyse the inverted pendulum system is the modern control system analysis using the state space method. While conventional control theory is based on the input-output relationship, or transfer function, modern control theory is based on the description of system equations in terms of n first-order differential equations, which may be combined into a first-order vector-matrix differential equation.

4.1 State and Output equation

State model of a linear time-invariant system is a special case of the general time-invariant model[3-19]:

$$\dot{x}(t) = f(x(t), u(t)) \tag{4.1}$$

$$y(t) = g(x(t), u(t)) \tag{4.2}$$

Derivatives of each state variable now becomes a linear combination of states and inputs, i.e.,

State equation:

$$\dot{x} = Ax + Bu \tag{4.3}$$

Output Equation:

$$y = Cx + Du \tag{4.4}$$

where,

x = 4-vector (state vector)

y = 2-vector (output vector)

u = 1-vector (input vector)

A = 4x4 matrix – State matrix

B = 4x1 matrix – Input matrix

C = 2x4 matrix – Output matrix

D = 2x1 matrix - Direct transmission matrix

The state of the system is closely related to the degrees of freedom. In the single inverted pendulum system, the transfer functions: $\theta(s)/V(s)$ and $\alpha(s)/V(s)$ are independent of each other, indicating that the system has two degrees of freedom. Thus, the state vector are of the form:

$$x = \begin{bmatrix} \theta & \alpha & \dot{\theta} & \dot{\alpha} \end{bmatrix}^T \tag{4.5}$$

From the mathematical modelling of the system, the matrices **A**, **B**, **C** and **D** are formed as follows:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 52.78 & -0.66 & -0.65 \\ 0 & 97.98 & -0.65 & -1.21 \end{bmatrix} \tag{4.6}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 274.45 \\ 273.92 \end{bmatrix} \tag{4.7}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{4.8}$$

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{4.9}$$

The inverted pendulum is a regulator system i.e., it has no reference input. The controllers are designed in such a way that it brings the system to its steady state value (zero) in a finite time for any perturbation.

For designing the controllers, the system should possess two important properties:

Observability and Controllability. These have been described as below.

4.2 System Interaction

In the world of control engineering, there are a slew of systems available that need to be controlled. The task of a control engineer is to design controller and compensator units to interact with these pre-existing systems. However, some systems simply cannot be controlled (or, more often, cannot be controlled in specific ways). The concept of **controllability**[4-18] refers to the ability of a controller to arbitrarily alter the functionality of the system plant.

The state-variable of a system, x, represents the internal workings of the system that can be separate from the regular input-output relationship of the system. This also needs to be measured, or observed. The term **observability** describes whether the internal state variables of the system can be externally measured.

4.2.1 Controllability

A system is said to be controllable at time t_0 if it is possible by means of an unconstrained control vector to transfer the system from any initial state $x(t_0)$ to any another state in a finite interval of time. Controllability depends upon the system matrix **A** and the control influence matrix **B**.

The necessary and sufficient condition for full state controllability of a system is that the controllability matrix M_c (4.10) should have a rank equal to the order of the system (n).

$$M_c = \begin{bmatrix} B & AB & A^2B & A^3B & \cdots & A^{n-2}B & A^{n-1}B \end{bmatrix} \tag{4.10}$$

4.2.2 Observability

A system is said to be observable at time t_0 if, with the system in state $x(t_0)$, it is possible to determine this state from the observation of the output over a finite interval of time. Observability depends upon the system matrix **A** and the output matrix **C**.

The necessary and sufficient condition for full state observability of a system is that the observability matrix M_o (4.11) should have a rank equal to the order of the system

(n).

$$M_o = \begin{bmatrix} C^T & A^TC^T & (A^T)^2C^T & (A^T)^3C^T & \cdots & (A^T)^{n-2}C^T & (A^T)^{n-1}C^T \end{bmatrix} \tag{4.11}$$

4.2.3 Principle of Duality

Consider the following two systems :

S_1 :

$$\dot{x} = Ax + Bu \tag{4.12}$$

$$y = Cx \tag{4.13}$$

S_2 :

$$\dot{z} = A^*z + Cz \tag{4.14}$$

$$n = Bz \tag{4.15}$$

where,

A^* - conjugate transpose of A

B^* - conjugate transpose of B

C^* - conjugate transpose of C

The principle of duality states that the system S_1 is completely state controllable if and only if system S_2 is completely observable; and vice-versa!

Hence, the problem of observer design for a system is actually a problem of control design for its dual system.

4.3 Controller Design

In a regulator type control system, the controller regulates the output and maintains it at desired level as specified by some reference. When the reference is time varying, the controller will attempt to excite the system in such a way that the output will track

the reference as closely as possible. Usually, the control signal applied to the plant of a system is a function of the difference between the reference input and the actual output, commonly referred to as the error signal. For a real system, disturbances will act on plant. An effective control system should be insensitive to external disturbances and responsive to changing inputs.

For this project, the primary objective is to balance the pendulum[13] in its upright position i.e. to make the upright equilibrium asymptotically stable. To achieve asymptotic stability of the upright (unstable) equilibrium position, the following three approaches are taken:

(a) State feedback using LQR optimisation.

(b) State observer feedback controller.

(c) Dual - loop PID controller.

First, a pole is placed in the uncontrolled system by taking the state feedback. This process is optimised using the LQR technique[14 -15]. Then with the desired poles of this compensated system the state observer and PID controllers are designed. The next three sections focus on the design of each controller.

Chapter 5

State feedback control using LQR optimisation

Given the position of closed-loop poles of a system, the gain K of the feedback loop can be determined. In the pole placement technique, if the system is completely state controllable and all state variables are available for feedback, then poles of the closed-loop system may be placed at any desired locations by means of state feedback through an appropriate state feedback gain matrix K.

The gain matrix K is obtained in such a manner to optimize the control process. Linear Quadratic Regulator (LQR)[21] is one of the main results of the theory of optimal control which is concerned with operating a system at the minimum cost. In this theory, system dynamics are usually described by a set of linear differential equations and the cost in the control process is represented as a quadratic functional.

The plant is linear and the cost function J is quadratic. For this reason, the problem of determining the state feedback control which regulates the states to zero to minimize J is called the Linear Quadratic Regulator (LQR).

In the optimal regulator problem, given the system

$$\dot{x} = Ax + Bu \tag{5.1}$$

the matrix K is determined of the optimal control vector as shown in figure 5.1

$$u(t) = -Kx(t) \tag{5.2}$$

so as to minimize the performance index

$$J = \int_0^\infty (x^T Q x + u^T R u) dt \tag{5.3}$$

where Q is a positive-definite (positive semi-definite) real symmetric matrix and R is a positive-definite real symmetric matrix. With this assumption, the first integral term $x^T Q x$ first integral term $x^T Q x$ is always positive or zero and the second term $u^T Q u$ is always positive at each time t for all values of x and u. Note that the second term accounts for the expenditure of the energy of the control signals. The matrices Q and R determine the relative importance of the error and the expenditure of this energy. In terms of eigenvalues, the eigenvalues of Q should be non-negative and of R should be positive. Usually, Q and R are selected to be diagonal for convenience, thus some entries of Q will be positive with some possible zeros on its diagonal while all the entries of R must be positive[4]. Note that R is invertible.

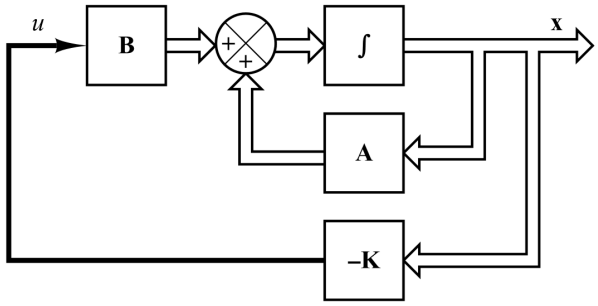


Figure 5.1: Closed loop control system with $u = -Kx$

The cost function J is a performance index of the cost of the whole control process and it can be interpreted as an energy function. The magnitude of the control action itself is included in the cost function so as to keep the cost, which is due to the control action itself, to be limited. Since both the state x and the control input u are weighted in J, if J is small, then both x and u are kept to be small. Furthermore, if J is minimized, then it is certainly finite, and since it is an infinite integral of x, this implies that x goes to zero as it goes to infinity, which guarantees that the closed-loop system will be stable.

By solving the algebraic Riccati equation(ARE)[18] for P

$$A^T P + PA + Q - PR^{-1}B^T P = 0 \tag{5.4}$$

the optimal LQR gain K is

$$K = R^{-1}B^T P \tag{5.5}$$

Using this K, the closed-poles of the compensated system is obtained. The minimal value of the performance criterion V using this gain is given by

$$V(x_0) = x^T P x_o \tag{5.6}$$

which only depends on the initial condition x_0. This mean that the cost of using the LQR gain can be computed from the initial conditions before the control is ever applied to the system.

5.1 LQR Control Method

The next step in designing such a control is to determine the feedback gains. In MATLAB the LQR function is used which gives the optimal controller. Using LQR function, two parameters i.e. R and Q can be chosen, which will balance the relative importance of the input.

The element at row 1, column 1 in Q matrix weights to the horizontal angle θ of the pendulum. Similarly the element at row 2, column 2 weights to the vertical angle α of the pendulum, element at row 3 column 3 weights to the angular velocity of the arm, element at row 4 column 4 weights to the angular velocity of the pendulum. R gives weight to the input voltage. The K matrix can be produced by choosing a suitable value of Q and R using MATLAB command. For a particular gain matrix the response of θ and α is plotted. Q and R matrix is adjusted by hit and trial method to obtain the desired response of the system.

By trial and error method Q and R is found to be

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{5.7}$$

$$R = [10] \tag{5.8}$$

This yields the feedback gains as

$$K = \begin{bmatrix} -0.3162 & 4.5266 & -0.2427 & 0.4090 \end{bmatrix} \tag{5.9}$$

and eigen values (or closed-loop poles) of the system as:

$$\lambda_1 = -1.5705 + 1.4220j \tag{5.10}$$

$$\lambda_2 = -1.5705 - 1.4220j \tag{5.11}$$

$$\lambda_3 = -22.0779 + 19.7110j \tag{5.12}$$

$$\lambda_4 = -22.0779 - 19.7110j \tag{5.13}$$

These closed-loop poles set the desired performance of the system.

With the help of Q and R, the K matrix i.e. the feedback matrix that will produce a good controller could be found by running the m-file code in MATLAB, and hence the response can be plotted easily.

Chapter 6

State Observer

6.1 State Observer

In the pole-placement[18] approach to the design of control systems, we assumed that all state variables are available for feedback. In practice, however, not all state variables are available for feedback. Then we need to estimate unavailable state variables. Here we show how, using the output and a model of the system, an estimate of the states can be obtained. A system that does this successfully is known as an ***observer***. If the state observer observes all state variables of the system, regardless of whether some state variables are available for direct measurement, it is called a ***full-order state observer***.

A state observer estimates the state variables based on the measurements of the output and control variables. State observers can be designed if and only if the observability condition is satisfied.

Consider the plant defined by

$$\dot{x} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{6.1}$$

$$y = \mathbf{C}\mathbf{x} \tag{6.2}$$

The observer is a subsystem to reconstruct the state vector of the plant. The mathematical model of the observer is basically the same as that of the plant, except that we include an additional term that includes the estimation error to compensate for inaccuracies in matrices **A** and **B** and the lack of the initial error.The estimation error or observation error is the difference between the measured output and the estimated output.The initial error is the difference between the initial state and the initial estimated state. Thus, we define the mathematical model of the observer to be

$$\begin{aligned}\dot{\tilde{\mathbf{x}}} &= \mathbf{A}\tilde{\mathbf{x}} + \mathbf{Bu} + \mathbf{K_e}(\mathbf{y} - \mathbf{C}\tilde{\mathbf{x}}) \\ &= (\mathbf{A} - \mathbf{K_e C})\tilde{\mathbf{x}} + \mathbf{Bu} + \mathbf{K_e y}\end{aligned} \tag{6.3}$$

where $\tilde{x}$ is the estimated state vector and $C\tilde{x}$ is the estimated output. The inputs to the observer are the output y and the control input u.Matrix Ke, which is called the observer gain matrix, is a weighting matrix to the correction term involving the difference between the measured output y and the estimated output $C\tilde{x}$. This term continuously corrects the model output and improves the performance of the observer. Figure 10–11 shows the block diagram of the system and the full-order state observer.

6.2 Full - Order State Observer

The order of the state observer that will be discussed here is the same as that of the plant. Assume that the plant is defined by Equations (6.1) and (6.2) and the observer model is defined by Equation (6.3).

To obtain the observer error equation, let us subtract Equation (6.3) from Equation (6.1) :

$$\begin{aligned}\dot{\mathbf{x}} - \dot{\tilde{\mathbf{x}}} &= \mathbf{Ax} - \mathbf{A}\tilde{\mathbf{x}} - \mathbf{K_e}(\mathbf{Cx} - \mathbf{C}\tilde{\mathbf{x}}) \\ &= (\mathbf{A} - \mathbf{K_e C})(\mathbf{x} - \tilde{\mathbf{x}})\end{aligned} \tag{6.4}$$

Let the difference between x and $\tilde{x}$ be e, or

$$\mathbf{e} = \mathbf{x} - \tilde{\mathbf{x}} \tag{6.5}$$

Then Equation (6.4) becomes

$$\dot{\mathbf{e}} = (\mathbf{A} - \mathbf{K_e C})\mathbf{e} \tag{6.6}$$

From Equation (6.6), we see that the dynamic behavior of the error vector is determined by the eigenvalues of matrix $A - K_eC$. If matrix $A - K_eC$ is a stable matrix, the error vector will converge to zero for any initial error vector $e(0)$. That is, will converge to x(t) regardless of the values of $x(0)$ and $\tilde{x}(0)$. If the eigenvalues of matrix $A - K_eC$ are chosen in such a way that the dynamic behavior of the error vector is asymptotically stable and

is adequately fast, then any error vector will tend to zero (the origin) with an adequate speed.

If the plant is completely observable, then it is possible to choose matrix K_e such that $A - K_eC$ has arbitrarily desired eigenvalues. That is, the observer gain matrix K_e can be determined to yield the desired matrix $A - K_eC$.

6.3 Building observer - based controllers

Consider the completely state controllable and completely observable system defined by the equations :

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \tag{6.7}$$

$$\mathbf{y} = \mathbf{Cx} \tag{6.8}$$

For the state-feedback control based on the observed state $\tilde{x}$ as shown in figure 6.1,

$$u = -\mathbf{K}\tilde{\mathbf{x}} \tag{6.9}$$

With this control, the state equation becomes

$$\dot{\mathbf{x}} = \mathbf{Ax} - \mathbf{BK}\tilde{\mathbf{x}} = (\mathbf{A} - \mathbf{BK})\mathbf{x} + \mathbf{BK}(\mathbf{x} - \tilde{\mathbf{x}}) \tag{6.10}$$

Substitution of the error vector e into Equation (6.10) gives

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{BK})\mathbf{x} + \mathbf{BKe} \tag{6.11}$$

Combining Equations (6.6) and (6.11), we obtain

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{e}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BK} & \mathbf{BK} \\ \mathbf{0} & \mathbf{A} - \mathbf{K_eC} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{e} \end{bmatrix} \tag{6.12}$$

From Equation (6.3), the state observer controller is thus :

$$\dot{\tilde{\mathbf{x}}} = [\mathbf{A} - \mathbf{BK} - \mathbf{K_e C}]\tilde{\mathbf{x}} + \mathbf{K_e y} \tag{6.13}$$

$$u = -\mathbf{K}\tilde{\mathbf{x}} \tag{6.14}$$

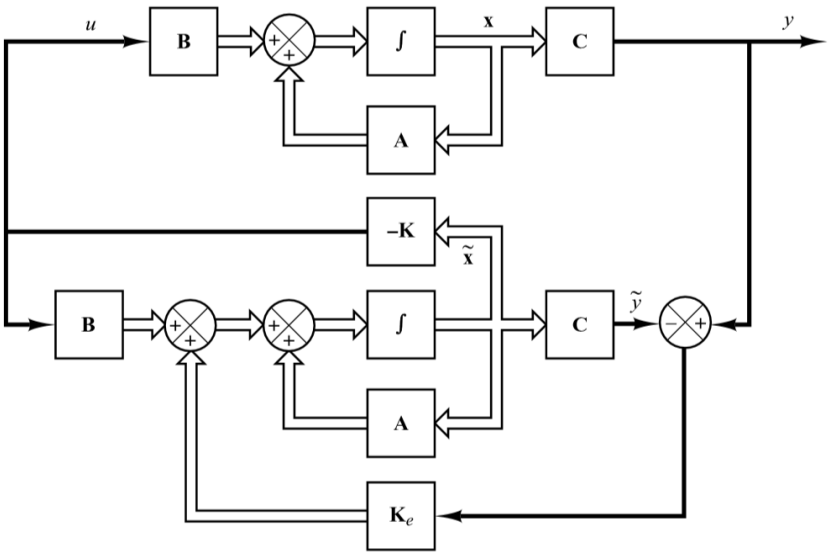


Figure 6.1: Observed-state feedback control system

In the practical implementation of the state observer, since a full-order state observer is being designed, the output matrix C is taken as identity matrix of order 4. the observer poles(E_1) are placed at 100 times the dominant close-loop poles (λ) on the real axis, i.e.,

Owing to the principle of duality (mentioned in section 4.2.3), the dual of the state observer system is a completely controllable system. Thus, the observer poles can be placed using the Ackermann's formula. In MATLAB, this is done easily using the *place* or *acker* command.

Desired observed poles of the system as:

$$\mu_1 = -2207.7 \tag{6.15}$$

$$\mu_2 = -2207.7 \tag{6.16}$$

$$\mu_3 = -157 \tag{6.17}$$

$$\mu_4 = -157 \tag{6.18}$$

From this, the observer gain is calculated to be as:

$$K_e = \begin{bmatrix} 2207.7 & 0 & 1 & 0 \\ 0 & 2207.7 & 0 & 1 \\ 0 & 52.8 & 156.4 & -0.6 \\ 0 & 98 & -0.6 & 155.8 \end{bmatrix} \tag{6.19}$$

Chapter 7

PID Controller Design

PID[14] (Proportional, Integral and Differential) controller is the most common form of feedback. It became the standard tool when process control emerged in 1940s. In PID controller the basic idea is the examination of signals from sensors placed in the system, called feedback signals.

$$K_P + \frac{K_I}{s} + K_D s = \frac{K_P s + K_I + K_D s^2}{s} \tag{7.1}$$

K_P= Proportional Gain

K_I= Integral Gain

K_D= Differential Gain

The error signal is sent to the PID controller, and the controller computes both the derivative and integral of the error signal. The signal (u) just past the controller is now equal to the proportional gain (K_P) times the magnitude of the error plus the integral gain (K_I) times the integral of the error plus the derivative gain (K_D) times the derivative of the error.

$$u = K_P e + K_I \int e(t)dt + K_D \frac{de}{dt} \tag{7.2}$$

The controller takes the new error signal and computes it derivative and integral again. This process goes on and on

7.1 The characteristics of P, I and D controllers

A proportional controller (K_P) will have effect of reducing the rise time, but never eliminates the steady-state error. An integral controller (K_I) will reduce the steady-state error but may make the transient response worse. A derivative controller (K_D) will have an effect on stability of the system, it reduces the overshoot, and improving the transient response. Effects of these three controllers can be summarized as shown in the table

CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S S ERROR
K_P	Decrease	Increase	Small Change	Decrease
K_I	Decrease	Increase	Increase	Eliminate
K_D	Small Change	Decrease	Decrease	Small Change

7.2 Dual loop PID control

There are two PID controllers one in each loop of the system , C1 is used to control theta and C2 is used to control alpha as shown in the schematic diagram below (figure 7.1.

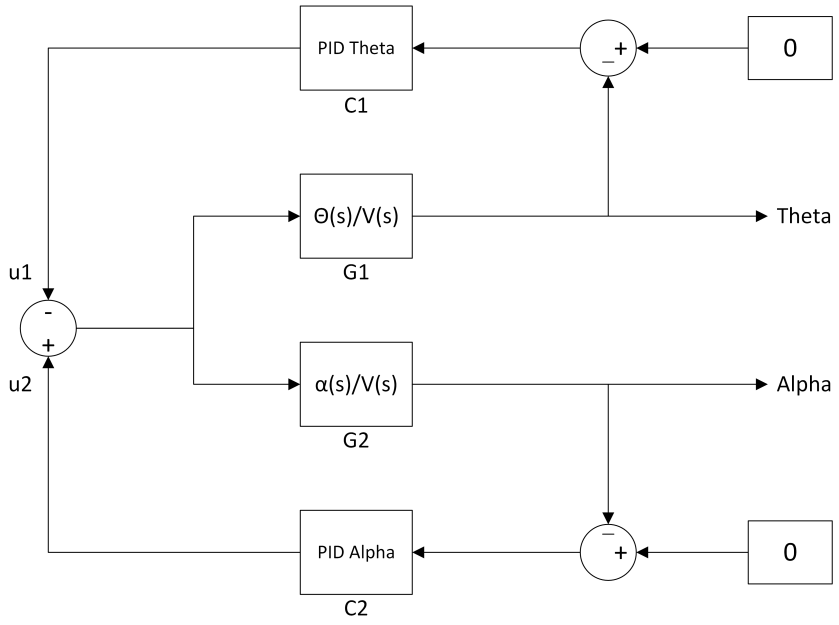


Figure 7.1: Schematic diagram of dual-loop PID controller

Here the initial conditions for pendulum's angle is $\alpha(0) = 0$ and pendulum's arm position $\theta(0) = 0$.

From the figure,

$$u = -u1 + u2 \tag{7.3}$$

$$u = -C1G1u + C2G2u = 0 \tag{7.4}$$

$$u(1 + C1G1u - C2G2u) = 0 \tag{7.5}$$

The equation – is known as the closed loop equation, where,

$$C1 = K_{P1} + \frac{K_{I1}}{s} + K_{D1}s \tag{7.6}$$

$$C2 = K_{P2} + \frac{K_{I2}}{s} + K_{D2}s \tag{7.7}$$

$$G1 = \frac{\theta(s)}{v_i(s)} = \frac{274.5s^2 + 154s + 12440}{s^4 + 1.87s^3 - 97.61s^2 - 30.37s} \tag{7.8}$$

$$G2 = \frac{\alpha(s)}{v_i(s)} = \frac{273.9s}{s^3 + 1.87s^2 - 97.61s - 30.37} \tag{7.9}$$

$$\begin{aligned} &s^5 + (1.87 + 273.9K_{D2} - 274.5K_{D1})s^4 + (-97.61 + 273.9K_{P2} - 154K_{D1} - 274.5K_{P1})s^3 \\ &+ (-30.37 + 273.9K_{I2} + 1240K_{D1} - 154K_{P1} - 274.5K_{I1})s^2 + (12440K_{P1} - 154K_{I1})s \\ &+ 12440K_{I1} = 0 \end{aligned} \tag{7.10}$$

The next step is to determine the PID gains for both pendulum's horizontal angle θ and vertical angle α. There are several methods for determining the PID gains. Here the Pole Placement method is adopted.

7.3 Pole Placement Method

The pole-placement method is somewhat similar to the root-locus method. The basic difference is that in the root-locus design we place only dominant closed loop poles (Poles

that are close to the imaginary axis) at the desired locations, while in the pole-placement design we place all closed-loop poles at desired locations.

Here we assume that all state variables are measurable and are available for feedback. The necessary and sufficient condition for pole-placement is that the system should be completely state controllable, such that the poles of the closed-loop system may be placed at any desired locations by means of state feedback through an appropriate state feedback gain matrix.

The present design technique begins with the determination of closed-loop poles based on the required transient response. From this closed-loop poles PID gains are obtained. The m-file for obtaining the closed-loop poles is shown in Appendix A2. Running this m-file we get the closed loops poles as

$$\lambda_1 = -1.5705 + 1.4220j \tag{7.11}$$

$$\lambda_2 = -1.5705 - 1.4220j \tag{7.12}$$

$$\lambda_3 = -22.0779 + 19.7110j \tag{7.13}$$

$$\lambda_4 = -22.0779 - 19.7110j \tag{7.14}$$

Form these four closed-loop poles it is clear that λ_1 and λ_2 are dominant poles. As from equation (7.10) the system should have five poles, let's assume another pole in such a way that it will not affect the system response and the best way is to choose λ_5 as 6 times the real part of the dominant pole, i.e. $\lambda_5 = -10$.

So, the characteristic equation becomes

$$(s - \lambda_1)(s - \lambda_2)(s - \lambda_3)(s - \lambda_4)(s - \lambda_5) = 0 \tag{7.15}$$

Putting all the values of λ in the above equation and solving it,

$$s^5 + 57.3s^4 + 1492s^3 + 13140s^2 + 33432s + 39320 = 0 \tag{7.16}$$

Comparing equation (7.10) and (7.16), and choosing K_{D1} as 0

$$K_{P1}=2.7265 \quad K_{P2}=8.5362$$
$$K_{I1}=3.1607 \quad K_{I2}=52.785$$
$$K_{D1}=0 \quad K_{D2}=0.2023$$

With this PID gains a PID controller is designed in Simulink.

Chapter 8

Simulation and Experimental Results

8.1 Introduction

This chapter discusses the simulation and experimental results of the output responses of the inverted Pendulum system with LQR, state observer and PID controllers. The experimental results are obtained by implementing the controllers on the Quanser SRV02 Rotary Inverted Pendulum.

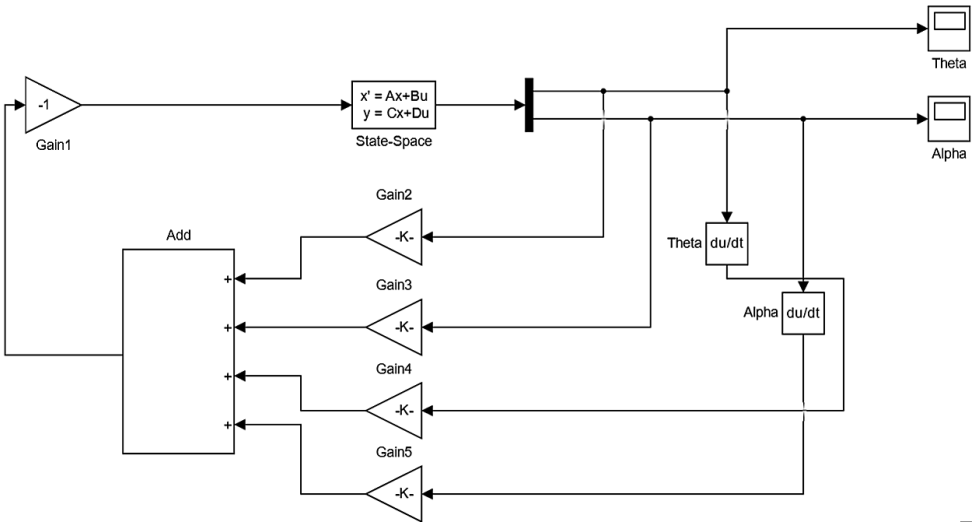
8.2 LQR Method

8.2.1 Simulation Results

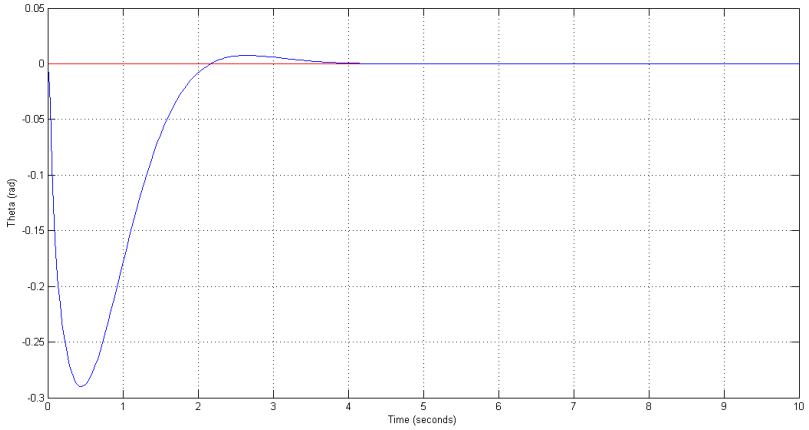
As discussed in section 5.1, the gain matrix of the state feedback control is determined by solving the algebraic Riccati equation. This, in MATLAB, is done easily using the command *lqr* [appendix]. These gain values are fed into the system described by the SimuLink schematic (figure 8.1a). The output of the system at the scope is shown in (figure 8.1b and 8.1c).

8.2.2 Experimental Results

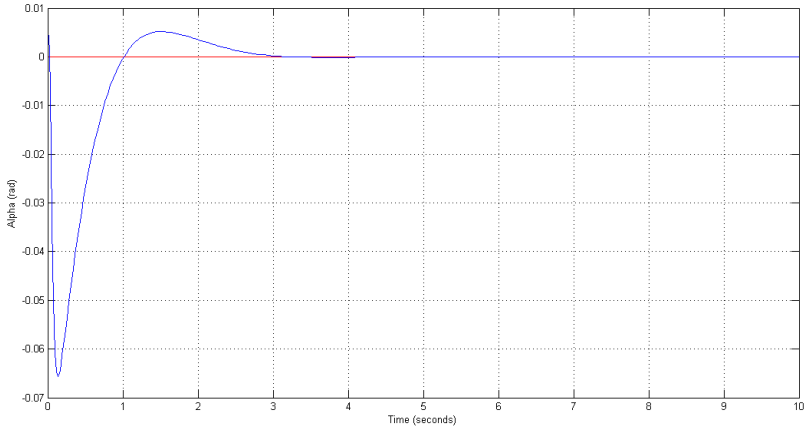
The LQR controller is then implemented using the Quanser set-up. Here the control voltage vs time (figure 8.2c) and the real-time variation of θ and α with respect to time is shown in figure 8.2a and figure 8.2b.



(a) Schematic diagram of the State feedback controller (using LQR) in Simulink

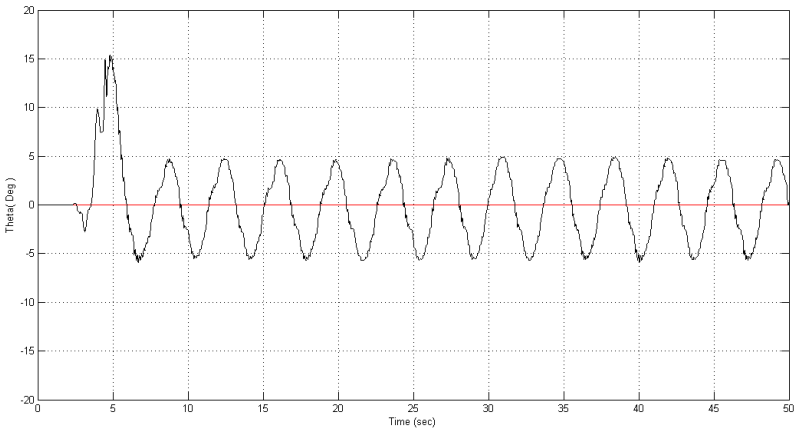


(b) LQR: Response curve of θ (simulation)

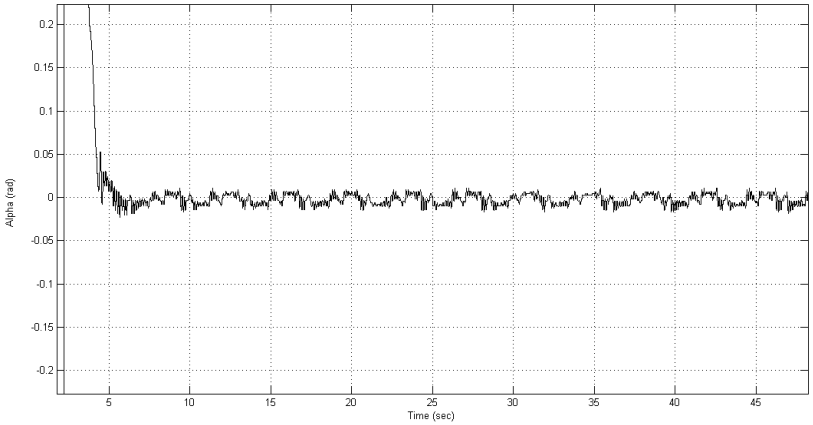


(c) LQR: Response curve of α (simulation)

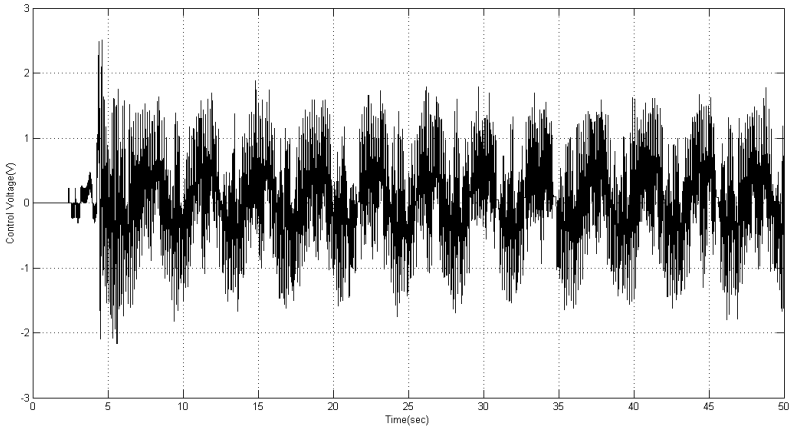
Figure 8.1: LQR : Simulation Results



(a) LQR: Response curve of θ (experimental)



(b) LQR: Response curve of α - magnified (experimental)



(c) LQR: Control voltage vs Time (experimental)

Figure 8.2: LQR : Experimental Results

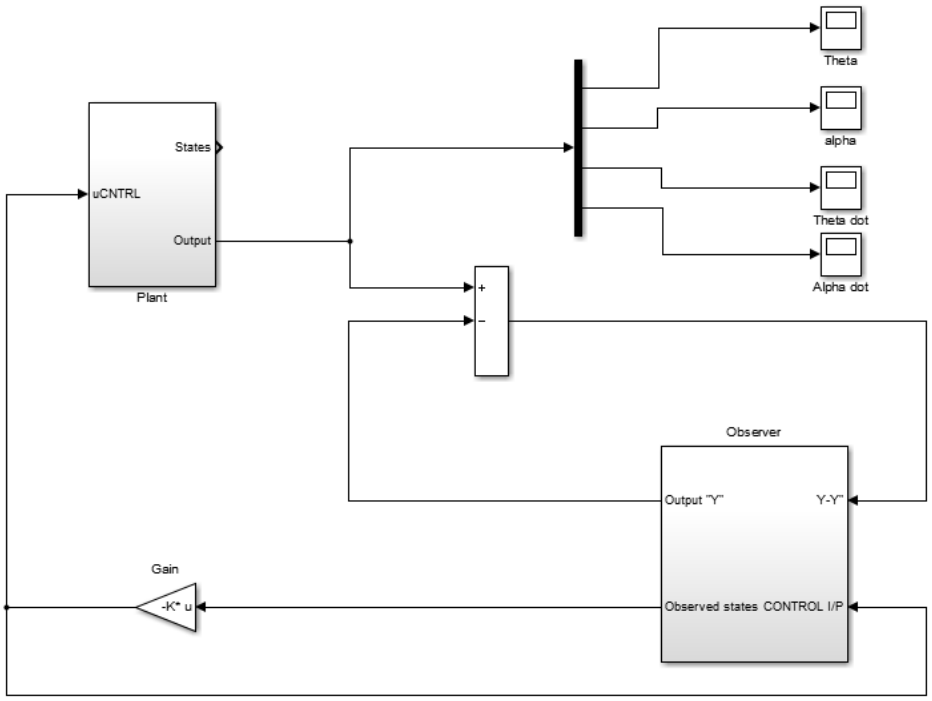
8.3 State observer

8.3.1 Simulation Results

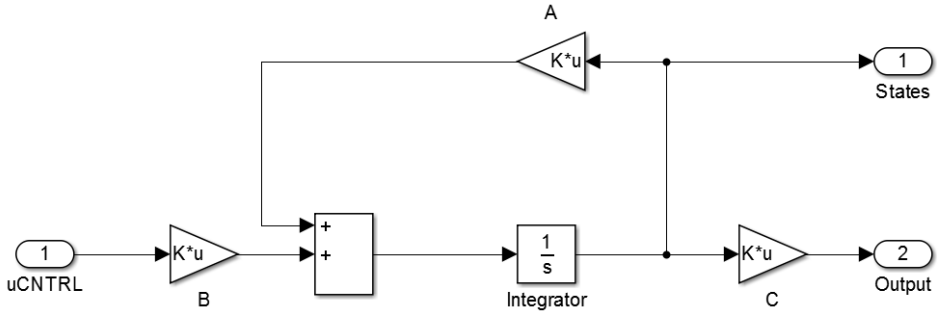
The procedure for calculating the gain matrix and observer poles is described in section [obs]. In MATLAB, this is done by placing the poles of the dual of the observer state equation at the desired location and finding the gain matrix using the commands *place* or *acker*. These gain values are fed into the system described by the SimuLink schematic (figure 8.3a). The output of the system at the scope is shown in (figure 8.4a and 8.4b).

8.3.2 Experimental Results

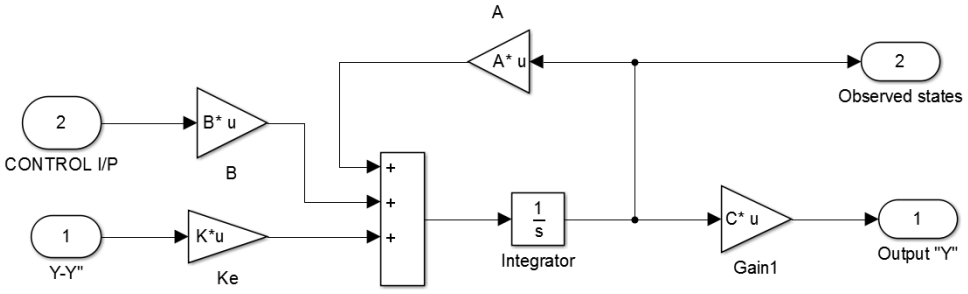
The observer is now implemented on hardware. Again, the control voltage vs time (figure 8.5b) and the real-time variation of θ and α with respect to time is shown in figure 8.5a and 8.5b.



(a) Schematic diagram of the State Observer in Simulink

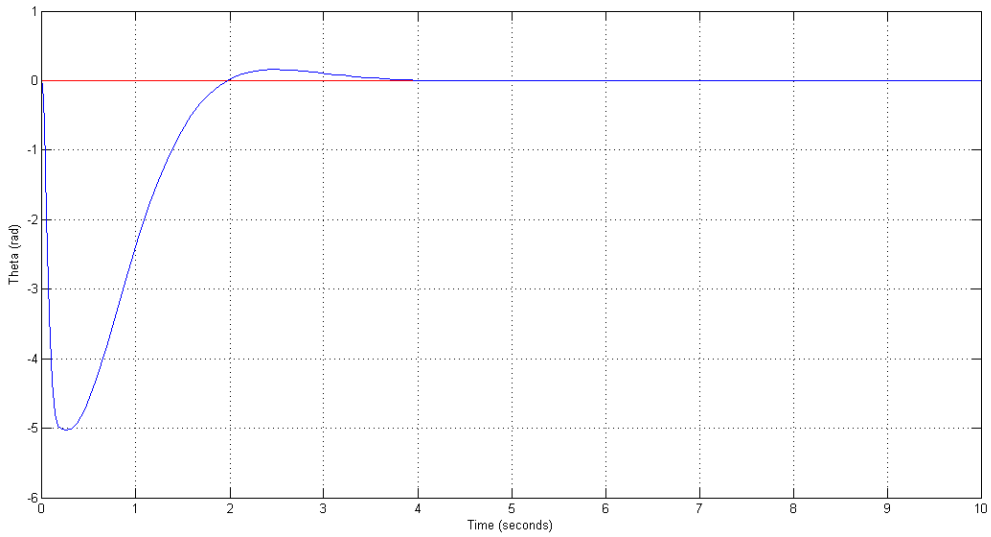


(b) Schematic diagram of inverted pendulum (inside plant block)

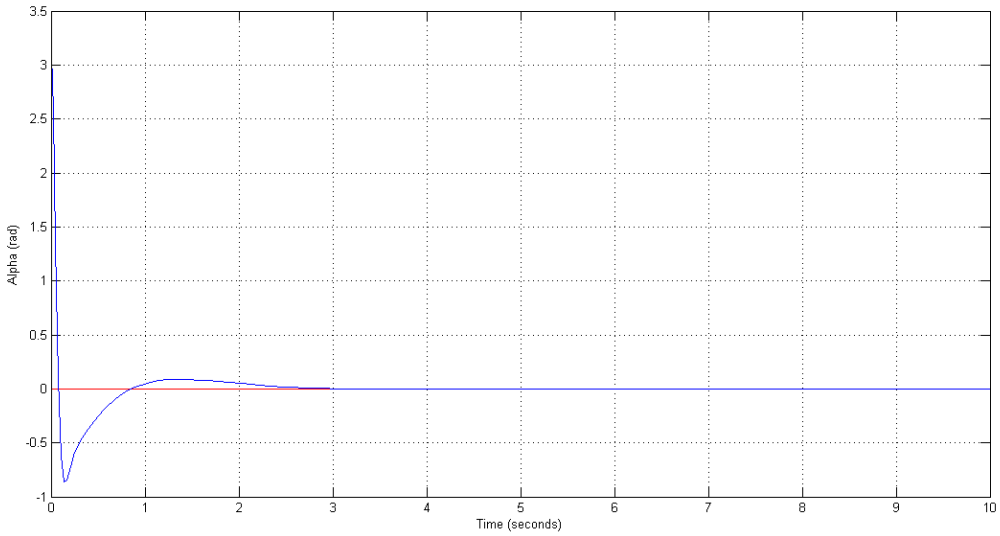


(c) Schematic diagram of Observer plant (inside observer block)

Figure 8.3: Full-Order State plant

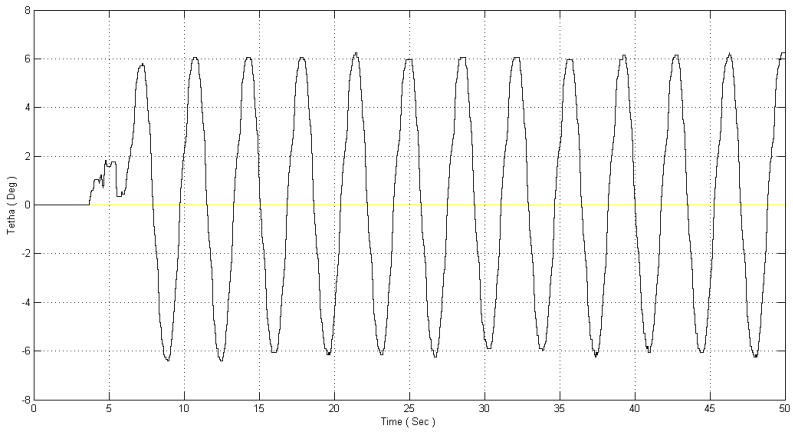


(a) State Observer: Response curve of θ (simulation)

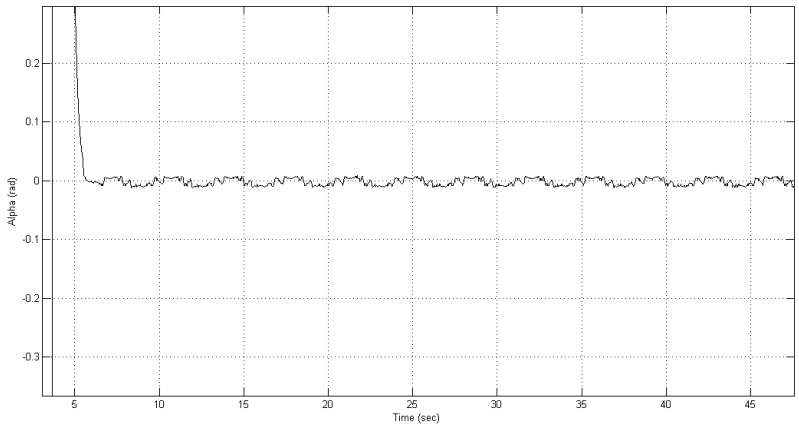


(b) State Observer: Response curve of α (simulation)

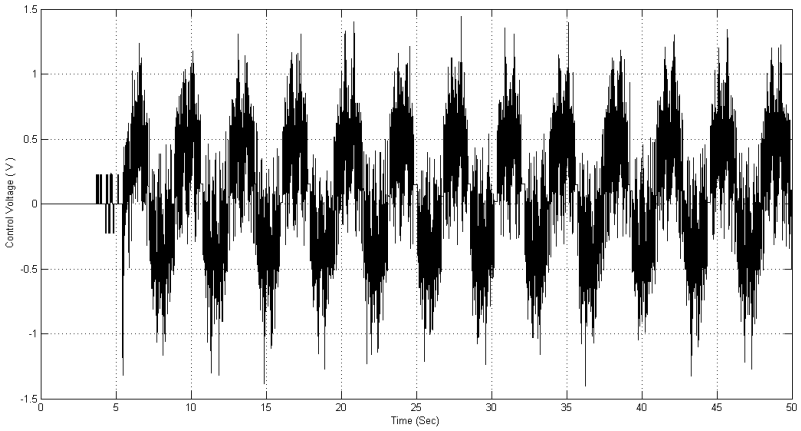
Figure 8.4: State Observer : Simulation Results



(a) State Observer: Response curve of θ (experimental)



(b) State Observer: Response curve of α - magnified (experimental)



(c) LQR: Control voltage vs Time (experimental)

Figure 8.5: State Observer : Experimental Results

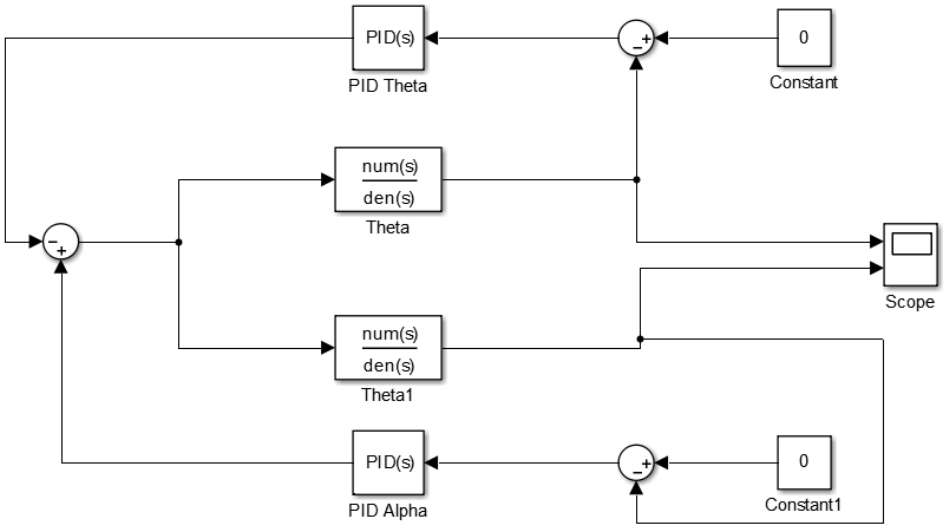
8.4 Dual-Loop PID

8.4.1 Simulation Results

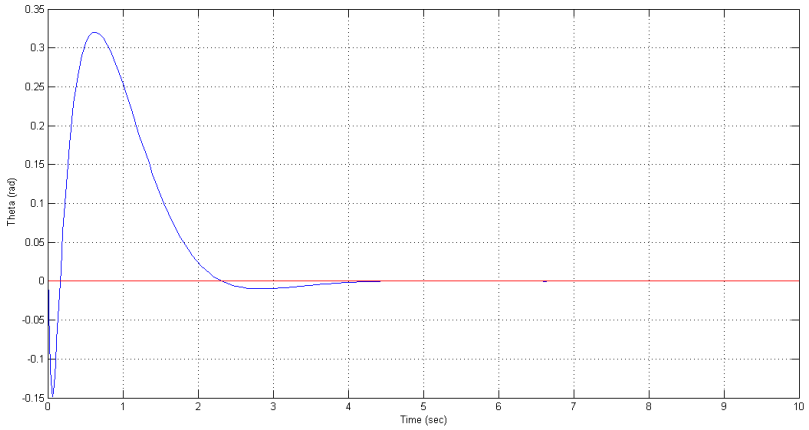
The tuning of the PID gain values is described in section [PID]. These gain values are fed into the system described by the SimuLink schematic (figure 8.6a). The output of the system at the scope is shown in (figure 8.6b and 8.6c).

8.4.2 Experimental Results

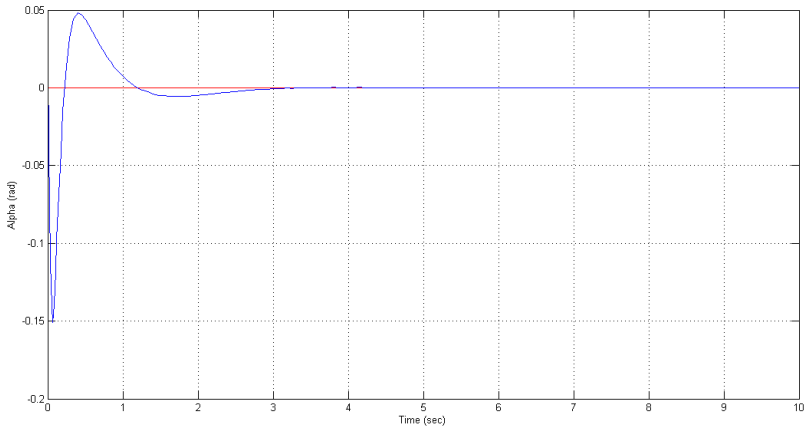
The Dual-loop PID is now implemented on inverted pendulum set-up. Again, the respective control voltage vs time (figure 8.7c) and the real-time variation of θ and α with respect to time is shown in figure 8.7a and figure 8.7b. It should be noted here that the oscillations in θ and α plots is due to the ambient noise and disturbance to the system.



(a) Schematic diagram of dual loop PID Controller in Simulink

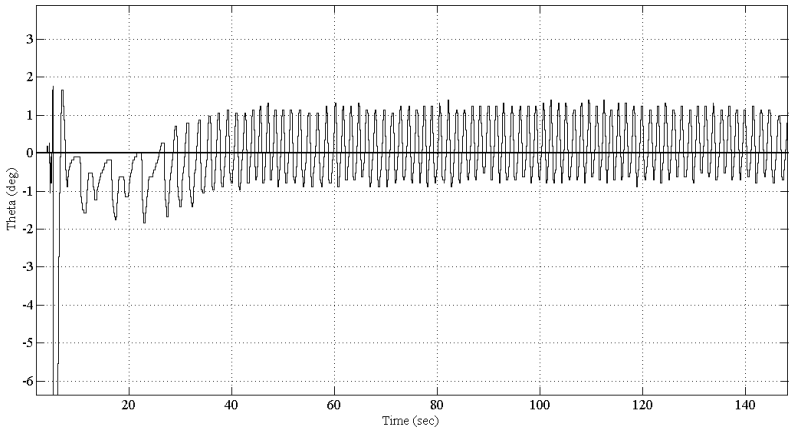


(b) PID Controller: Response curve of θ (simulation)

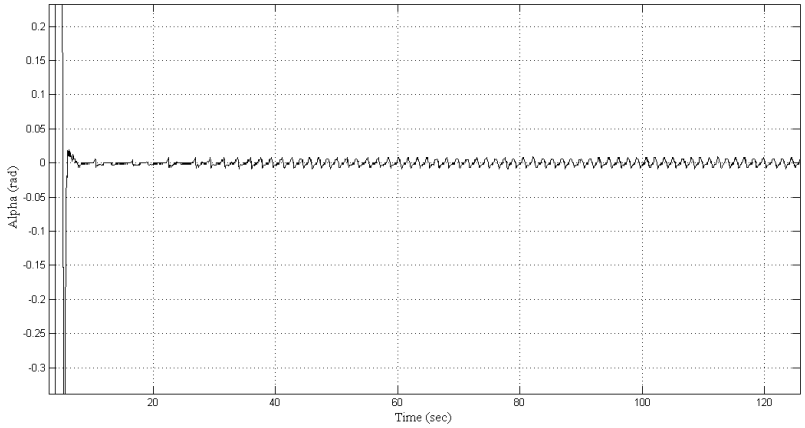


(c) PID Controller: Response curve of α (simulation)

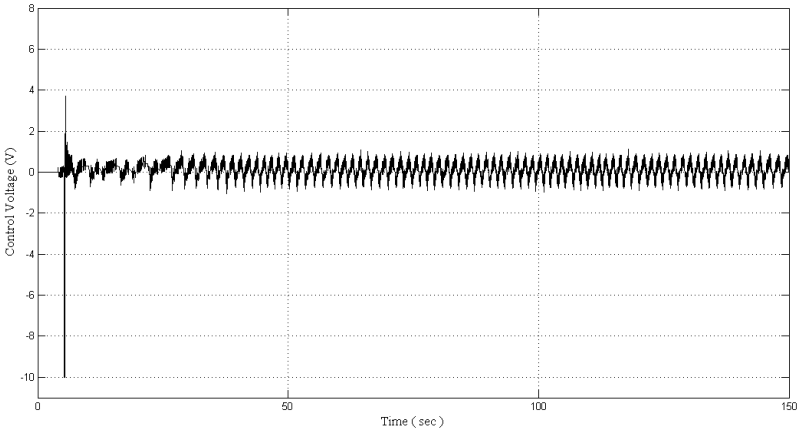
Figure 8.6: PID Controller : Simulation Results



(a) PID COntroller: Response curve of θ - magnified (experimental)



(b) PID Controller: Response curve of α - magnified (experimental)



(c) PID Controller: Control Voltage vs Time (experimental)

Figure 8.7: PID Controller : Experimental Results

Chapter 9

Conclusion

The inverted pendulum is a classic problem in control theory. It is used as a benchmark for testing control strategies (PID controllers, state space representation, neural networks, fuzzy control, genetic algorithms, etc.). It is inherently an unstable but completely controllable system. This serves as the motivation for designing the controller at the upright position. The controllers is designed in such a manner that it must maintain the pendulum angle(α) at 0° to the vertical axis and the horizontal arm angle is maintained at 0° to its reference. This is termed as the robustness of the system under controller action. From the simulation and experimental results, it can be observed that PID controller gives a better robustness compared to LQR controller and state observer. The state observer was found to be most sluggish.

	LQR	Observer	PID
Theta(deg)	-5 to 5	-6 to 6	-1 to 1.5
Alpha(deg)	-1.4 to 0.85	-1.14 to 0.75	-0.57 to 0.57
Control Signal(V)	-1.8 to 1.8	-1.5 to 1.5	-1 to 1

The above experimental results shows that the oscillation of the α and θ about 0° is least in PID controller and most in the state observer. This is in accordance with the simulation results that the PID Controller is most robust and observer is most sluggish. This is the main reason as to why PID controllers are preferred in industrial applications.

Chapter 10

Appendix

MATLAB CODES

State observer design

```
A = [0 0 1 0;0 0 0 1;0 52.78 -0.66 -0.65;0 97.98 -0.65 -1.21];
B = [0;0;274.45;273.92];
C = eye(4);
K = [-0.3162 4.5266 -0.2427 0.4090];
E = eig(A-B*K);
E1 = real(E);
Ke = (place(A',C',E1))';
sys=ss([A-B*K B*K; zeros(4,4) A-Ke*C],eye(8),eye(8),eye(8));
t=0:0.001:4;
z=initial(sys,[0;pi;0;0;0;pi;0;0],t);

x1=[1 0 0 0 0 0 0 0]*z';
x2=[0 1 0 0 0 0 0 0]*z';
x3=[0 0 1 0 0 0 0 0]*z';
x4=[0 0 0 1 0 0 0 0]*z';
e1=[0 0 0 0 1 0 0 0]*z';
e2=[0 0 0 0 0 1 0 0]*z';
e3=[0 0 0 0 0 0 1 0]*z';
e4=[0 0 0 0 0 0 0 1]*z';

subplot(3,3,1);  plot(t,x1); grid;
ylabel('theta');  xlabel('t(sec)');

subplot(3,3,2);  plot(t,x2); grid
ylabel('alpha');  xlabel('t(sec)');

subplot(3,3,3);  plot(t,x3); grid
ylabel('theta-dot');  xlabel('tsec)');

subplot(3,3,4);plot(t,x4);  grid

ylabel('alpha-dot');xlabel('t(sec)');
```

subplot(3,3,5);plot(t,e1); grid

ylabel('theta observer error'); xlabel('t(sec)');

subplot(3,3,6);plot(t,e2); grid

ylabel('alpha observer error'); xlabel('t(sec)');

subplot(3,3,7);plot(t,e3); grid

xlabel('theta-dot observer error');ylabel('t(sec)');

subplot(3,3,8);plot(t,e4); grid

xlabel('alpha-dot observer error'); ylabel('t(sec)');

To implement LQR control method in rotary inverted pendulum
A = [0 0 1 0;0 0 0 1;0 52.78 -0.66 -0.65;0 97.98 -0.65 -1.21];
B = [0;0;274.45;273.92];
C = [1 0 0 0; 0 1 0 0;0 0 1 0;0 0 0 1];
Q = [1 0 0 0; 0 100 0 0;0 0 0 0;0 0 0 0];
R = [10];
$[K, S, E] = lqr(A, B, Q, R)$;
sys = ss(A-B*K,eye(4),eye(4),eye(4));
t = 0:0.01:8;
x0 = [0;0;0;1];
x = initial(sys,x0,t);
x1 = [1 0 0 0]*x';
x2 = [0 1 0 0]*x';
x3 = [0 0 1 0]*x';
x4 = [0 0 0 1]*x';

subplot(2,2,1);plot(t,x1);grid
xlabel('t(sec)'); ylabel('theta')

subplot(2,2,2);plot(t,x2);grid
xlabel('t(sec)'); ylabel('alpha')

subplot(2,2,3);plot(t,x3);grid
xlabel('t(sec)'); ylabel('theta-dot')

subplot(2,2,4);plot(t,x4);grid
xlabel('t(sec)'); ylabel('alpha-dot')

NOMENCLATURE

The following are the list of variables used in this thesis :

m_p	Mass of the pendulum (kg)
L_p	Length of the pendulum (m)
J_p	Pendulum moment of inertia about center of mass.($kg.m^2$)
B_p	Pendulum viscous damping coefficient
m_r	Mass of rotary arm with two thumb screws.(kg)
L_r	Rotary arm length from pivot to tip.(m)
J_r	Rotary arm moment of inertia about pivot($kg.m^2$)
B_r	Arm viscous damping coefficient
θ	Arm angular displacement.(deg)
α	Pendulum angular displacement from vertical plane.(deg)
τ	Torque induced by SRV02
η_g	Gearbox efficiency
K_g	High gear total gear ratio
η_m	Motor efficiency
K_t	Motor current-torque constant(Nm/A)
K_m	Motor back emf constant
R_m	Motor armature resistance(Ω)

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